Reevaluating the Relationship Between Transferable Property Rights and Command-and-Control Regulation

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Executive Summary

Economists generally assume that systems of transferable property rights are preferable to non-market systems. This paper suggests that the design of a market-based policy that dominates a command-and-control regime is more subtle than is commonly believed, even in theory. The subtlety arises because identical approaches to monitoring and enforcement will not generally yield the same results in different regulatory environments. The paper identifies conditions under which a kind of market dominance result obtains. The theory is then applied to the problem of trading rights to emit pollutants from motor vehicles.

Reevaluating the Relationship Between Transferable Property Rights and Command-and-Control Regulation

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1. Introduction

A commonly held view in the economics profession suggests that it is possible to design systems of transferable property rights that are more efficient than systems that do not allow transferability. Take, for example, the case of the environment. Suppose that the government currently specifies that firms use a particular pollution control technology to achieve an industry-wide emissions target of 100 tons of pollution. Most economists would view such an approach as unnecessarily wasteful. For if 100 transferable property rights were issued, each of which allowed for up to one ton of pollution to be emitted, a market among polluters could emerge that would encourage a less costly approach to achieving the desired environmental objective (see, e.g., Dales, 1968). The key to the success of this market is that it provides individual firms with greater flexibility in achieving a prescribed social objective along with an incentive to develop environmental innovations that are less costly and more effective.

Recently, the presumed superiority of transferable property rights approaches has been questioned on both theoretical and practical grounds. One theoretical critique relates to the possibility of input or output markets that are not structurally competitive. For example, a firm may have market power in the transferable rights market, and thus be able to behave as a monopolist or a monopsonist in that market (Hahn, 1984). Alternatively, a firm may be able to use its advantage in a transferable property rights market to gain a strategic advantage in an output market, which could have adverse impacts on welfare (Misiolek and Elder, 1989; Malueg, 1990). A second critique relates to the possibility that a regulatory regime based on technological standards can result in overcontrol, and thus can result in higher net benefits than a transferable property rights approach (Oates, Portney and McGartland, 1989). These critiques, while interesting, do not

fundamentally question the ability of market instruments to achieve a specified objective at least cost when there is perfect competition.

The practical critique of markets in transferable property rights usually relates to their implementation, particularly in the environmental arena. It is frequently alleged by critics of environmental markets that such markets provide firms with an additional mechanism for circumventing pollution control regulations. This critique, while true in selected instances, does not appear to be a valid characterization of the performance of environmental markets to date (Hahn, 1989). Nonetheless, politicians and environmental groups persist in using this critique to slow the introduction of markets in transferable property rights for controlling externalities.

This paper reexamines the theoretical institutional design problem and then applies the theory to the regulation of pollution from motor vehicles. The theoretical presentation is quite general. In principal, it applies to any problem in which perfect enforcement of property rights is not costless. In practice, it applies to problems in which monitoring and enforcement are difficult or costly.

Section 2 develops a theoretical model that shows why the design of a transferable property rights system that dominates a non-transferable rights system is more subtle than is commonly believed, even for a highly stylized case in which there is perfect competition. The subtlety arises because identical approaches to monitoring and enforcement will not generally yield the same results in different regulatory environments. The primary theoretical contribution of the paper is to define conditions under which a kind of market dominance result obtains. Unfortunately, these conditions will not generally be satisfied, and in any case, require a great deal of information on the part of the decision maker. Section 3 examines the implications of the model for the design of markets. An application of the model is presented in section 4. Section 5 reviews the principal results and suggests areas for future research.

2. The Theoretical Model

There is a voluminous literature that examines how firms and individuals respond to different institutions with incomplete enforcement (see, e.g., Becker, 1968, Harford, 1978, Viscusi and Zeckhauser, 1979, and Jones and Scotchmer, 1990). This research focuses primarily on the impact of imperfect monitoring and enforcement on a given institution. In contrast, we are interested in comparing responses across two institutions. Thus, while we use a similar framework in examining these issues, the focus is different.

The theoretical model is based on a standard model of pollution control, but is more general in the sense that it applies to a wide range of problems where the government or a central agent regulates output. In the first case, the government announces a standard, S, which the firm must meet or else suffer the consequences. The firm then has to choose whether and how it will meet the standard. It can react to the standard by producing some amount of control, A. The firm minimizes the cost of control plus the expected costs of falling short of the standard for each unit produced. The average cost of control is designated by C(A) while the consequences of violating the standard are given by the average expected penalty function, F(-A-S).

In the simplest case, the firm will:

$$\underset{A}{\text{Minimize}} \quad C(A) + F(-A-S).$$

subject to: $A \geq 0$ and $S \geq 0$.

The form of the cost and expected penalty functions requires some elaboration. The cost function is assumed to be strictly convex and twice differentiable so that C'>0 and C">0, where "'" denotes the first derivative. The expected penalty function's argument is presumed to be a function of -A-S, which is a proxy for the expected violation.[1] As the expected violation

[1]Say for example, A represents pollution abatement. Then actual emissions can be represented by the equation E = k - A, where k is a constant. The expected violation is then measured by E-S

increases, the expected penalty and marginal expected penalty are assumed to increase, so that $F'>0$ and $F''>0$. Harford (1978) shows that the assumption on F' is quite reasonable when the expected penalty function represents the product of a probability and fine, both of which are functions of the size of a violation. The assumption on F'', while plausible, does not immediately follow from his analysis. Our assumption on F'' is motivated by an interest in exploring interior solutions, which we believe are relevant for a wide range of practical problems.

A simple example will illustrate the nature of the model. When the Environmental Protection Agency announces that all cars must meet a standard of, say, S grams per mile, auto-makers select control strategies (e.g., a catalyst) in response to the standard. Firms will balance the direct costs of control with the expected penalty associated with different levels of control. Any given level of control, say A^*, will be more likely to pass federal compliance tests as the standard is relaxed (i.e., S is increased). This is the rationale behind the assumption that F' is positive.

First, consider the general "command-and-control" case where the government chooses a single output standard[2] in a "one-size fits all" approach.[3] Assume that there are n classes of objects (e.g., vehicles) subject to the regulation, with each class i having an average cost, $C_i(A_i)$, and an average expected penalty, $F(-A_i-S)$. Let n_i represent the number of units produced in class i divided by the total number of units produced in all classes, so that $\Sigma n_i = 1$.[4]

$= k - A - S$. Thus, the expected violation differs from the argument in the penalty function by an arbitrary constant.

[2]See Jones (1990) for a more detailed analysis of this case.

[3]The extreme variant of the command-and control approach is for the government to specify a particular technology that firms must use, sometimes referred to as a "technology-forcing" standard. For example, in the 1990 Clean Air Act Amendments, auto makers are required to install a canister to recover gasoline vapors (U.S. House of Representatives, 1990, Section 202, p. 79). An approach that can provide firms with more flexibility is to specify a performance standard, which does not explicitly require a specific technology. The modeling approach used here corresponds most closely to a performance standard. See Hahn (1989, Chapter 2) for a more detailed discussion of the nature of different regulatory approaches.

[4]Summations without indices are presumed to go from 1 to n.

The command-and-control problem can now be stated

(1) $$\underset{A_1,\ldots,A_n}{\text{Minimize}} \quad \sum n_i \, [C_i(A_i) + F(-A_i-S)],$$

subject to: $A_i \geq 0$, for $i = 1,\ldots,n$.

The first order conditions for an interior minimum are given by:

(2) $C_i'(A_i) - F'(-A_i-S) = 0$, for $i = 1,\ldots,n$.

Equation (2) says that the marginal cost for each category equals the marginal expected penalty for that category. Because costs typically differ across categories, marginal costs will not generally be equated.

The command-and-control problem can apply to a single firm or several firms. This follows from the fact that the value of A_i is not affected by the value of A_j for $j \neq i$. That is, it is possible to state (1) as n separate minimization problems with respect to the relevant i using the argument inside the summation sign. The solution will be invariant to the number of firms, provided that an entire class, i, is produced by a single firm, so that the cost functions do not change.

The preceding centralized approach can be compared to a decentralized approach in which the government allows a single agent to average over all n classes of objects in a particular way. For each class of objects, the firm can choose not only the control, A_i, but also the level of the performance standard, S_i, so long as the weighted average of the performance standards do not exceed the standard, S. Formally this yields:

(3) $$\underset{A_1,\ldots,A_n,S_1,\ldots,S_n}{\text{Minimize}} \quad \sum n_i \, [C_i(A_i) + F(-A_i-S_i)]$$

subject to: $S - \sum n_i S_i \geq 0$, and

$$A_i \geq 0, S_i \geq 0, \text{ for } i = 1,\ldots,n.$$

While the constraint relating S_i to S in (3) is written as an inequality, it is clear that it will be satisfied as an equality, since an increase in any performance standard, ceteris paribus, would decrease the objective function.

The solution to (3) can be characterized by defining the following Langrangian:

$$\text{(4)} \qquad \underset{A_1,\ldots,A_n,S_1,\ldots,S_n,\lambda}{\text{Minimize}} \quad \sum n_i \, [C_i(A_i) + F(-A_i-S_i)] + \lambda[S - \sum n_i \, S_i],$$

where λ is a Lagrange multiplier, corresponding to the cost of tightening the standard (i.e., decreasing S).

The first order conditions for an interior minimum are given by:

(5a) $C_i'(A_i) - F'(-A_i-S_i) = 0$, for $i = 1,\ldots,n$,

(5b) $F'(-A_i-S_i) + \lambda = 0$, for $i = 1,\ldots,n$, and

(5c) $S - \sum n_i \, S_i = 0$.

The objective function in (4) will be characterized as the "trading" problem. While strictly speaking, it is an averaging problem involving only one firm, it is possible to show that a solution to this problem is equivalent to a comparable solution to the trading problem in which there are multiple firms.[5]

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[5] The proof is sketched here. It follows directly from the work of Montgomery (1972). First, assume that there are n separate price-taking firms participating in a permit market. (Fewer than n firms poses no problems, but adds to notational complexity.). Each firm receives an initial endowment of permits, S, for each object. The firm must own L_i permits per object if it wishes to choose a performance standard of S_i or lower. It can purchase or sell permits at the market price, p. A market equilibrium to this problem is a set of (3n+1) nonnegative values, defined by A_i^*, S_i^*, L_i^*, and p, such that A_i^*, S_i^*, L_i^* minimize

$$C_i(A_i) + F(-A_i-S_i) + p\,(L_i - S)$$

Note that the trading problem uses the same expected penalty function as the command-and-control problem. This enables us to examine the conventional wisdom that under comparable systems of monitoring and enforcement, a transferable property rights solution can be guaranteed to achieve an equivalent target at the same or lower cost.

The first order conditions to the trading problem are somewhat different than the command-and-control problem. In particular, combining (5a) and (5b) yields the result that $C_i' = -\lambda$ for all i. This mimics the conventional result on markets in transferable property rights when there is perfect enforcement. Marginal costs for each class are set equal to the equilibrium permit price, or in the averaging case, to the marginal increase in costs resulting from a decrease in the performance standard. The reason that the result obtains in this more general setting is because the same expected penalty function applies to all classes. If this result were relaxed, then the conditions for efficiency would no longer require that marginal costs be equated across sources.[6]

Equation (5b) yields another key result that is stated in the following lemma:

Lemma 1: The proxy for the expected violation, $-A_i-S_i$, is equal across all categories.[7]

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subject to: $L_i \geq S_i$ for all i, and which also satisfy the market clearing conditions $S - \Sigma n_i L_i \geq 0$ and $p [S - \Sigma n_i L_i] = 0$.

A market equilibrium can be constructed from the averaging problem, and a solution to the averaging problem can be constructed from a market equilibrium as follows: Suppose there is a nonnegative solution, A_i, S_i, $-\lambda$ to the averaging problem defined by (4). Then, let $A_i^*=A_i$, $S_i^*= L_i^*=S_i$, and $p = -\lambda$ Conversely, if A_i^*, S_i^*, L_i^*, and p is a market equilibrium, then letting $A_i=A_i^*$, $S_i=S_i^*$, and $\lambda = -p$ will yield a solution to (4) (though not necessarily an interior solution where endogenous variables are strictly positive).

[6]For an insightful discussion of the effects of noncompliance on permit price and efficiency, see Malik (1990).

[7]In the example considered below, the control, A, will be thought of in terms of abatement or pollution control. Abatement is sometimes related to emissions as $A = E^O - E$, where E^O is some

Proof: The result follows immediately upon noting that in equilibrium, the marginal expected penalty equals the price of a permit. Because the expected penalty function for each class is the same, by assumption, the argument within each expected penalty function, $-A_i-S_i$, must be equal for (5b) to obtain.

The central theoretical challenge will be to relate the solutions to the trading problem (T) and the command-and-control problem (C). In general, the amount of control and the direct costs for C and T will be different. However, for the special case when the expected penalty function is linear, it can be shown that the two solutions will yield the same amount of control, A_i, for each class i.

Lemma 2: Let $F(\Delta) = a + b\,\Delta$. Then the amount of control and the direct cost of control for each class will be equal for the two problems.

Proof: Note that $F' = b$. This means that the command-and-control problem is really a taxation problem, with the marginal cost of control set equal to the effective tax, b. Equation (2) implies $C_i'(A_i) = b$ for $i = 1,\ldots,n$, but this is precisely the same condition as equation (5a) for this special case. Marginal costs of control are equated in both cases. The selection of particular performance standards in the trading problem is a matter of indifference because the value of the sum of the expected penalty functions does not vary so long as equation (5c) is satisfied.

The problem is less straightforward when the marginal expected penalty function varies with the amount of control. To help compare these solutions, it is instructive to develop a more precise characterization of the trading solution. Let $A(x) = \Sigma n_i C_i'^{-1}(x)$. Then define $C'(A) = A^{-1}(x)$.[8] Note that C' is a weighted aggregation of the marginal cost schedules, where the weights represent the proportion of objects in each class.

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baseline level of emissions. If the penalty function is, in fact, a function of the difference between E_i and S_i for each class i, then the lemma here will generally be true only for the case where E^o is constant across emissions sources.

[8]Recall that the ith marginal cost schedule C_i' is strictly increasing and differentiable. Thus, C' is also strictly increasing with a positive first derivative.

The solution to the trading problem can now be characterized as follows:

Lemma 3: An interior solution to the trading problem is completely characterized by the intersection of the weighted marginal cost curve, $C'(A)$, and the marginal expected penalty function, $F'(-A-S)$.

Proof: See appendix.

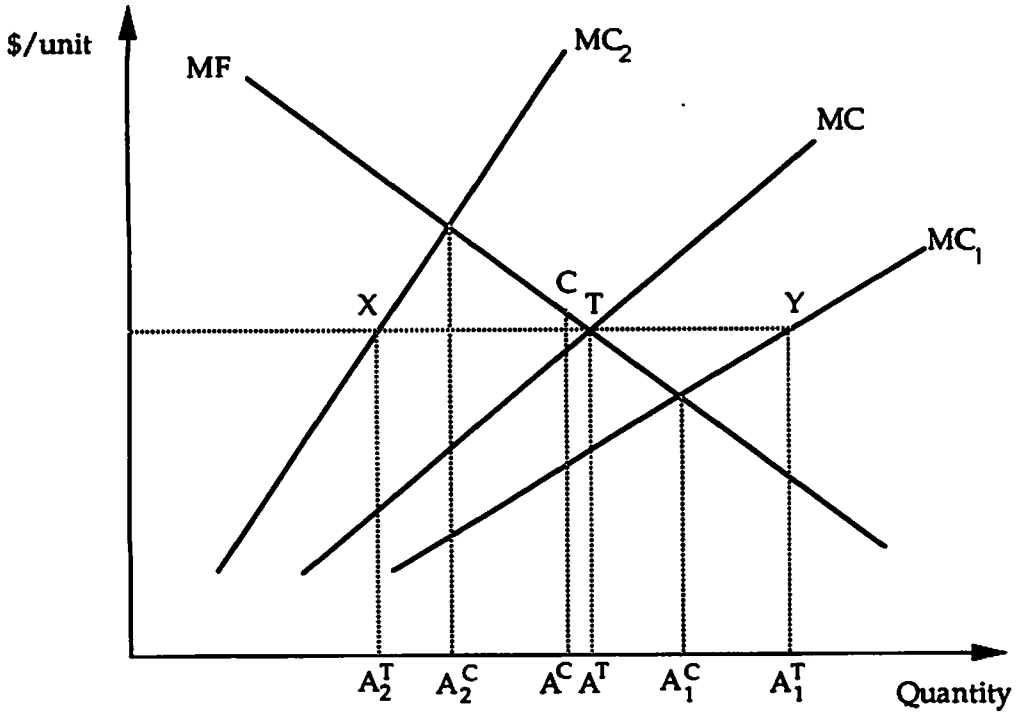
The general problem is to define conditions for the cost or the expected penalty function that will allow for categorical statements on how the amount of control under trading compares with that of command-and-control. To furnish some intuition for the theorem developed below, two cases are presented in which there are linear marginal cost and expected penalty functions. Figure 1 compares the average control under trading with the average control under command-and-control for two special cases. Variables with the superscript "T" are associated with trading and variables with the superscript "C" are associated with command-and-control. Average control is defined as $A = \Sigma n_i A_i$ with appropriate superscripts added to denote the particular institution.[9]

The solutions in Figure 1a and 1b require some further explanation. The two graphs are similar, except for the placement of the marginal cost curves, so it suffices to focus on Figure 1a. There are two classes (denoted by the subscripts 1 and 2) with equal numbers of objects, so that $n_1 = n_2 = .5$. The key to understanding the figures is to identify the equilibrium controls for the two problems. These are determined by the marginal cost curves, denoted by MC_1 and MC_2, and the marginal expected penalty curve, denoted by MF. Begin with the command-and-control problem. Assuming an interior minimum, equilibrium is determined by the intersection of MF with MC_1 and MC_2, yielding controls A_1^C and A_2^C. Average control, A^C, occurs at point

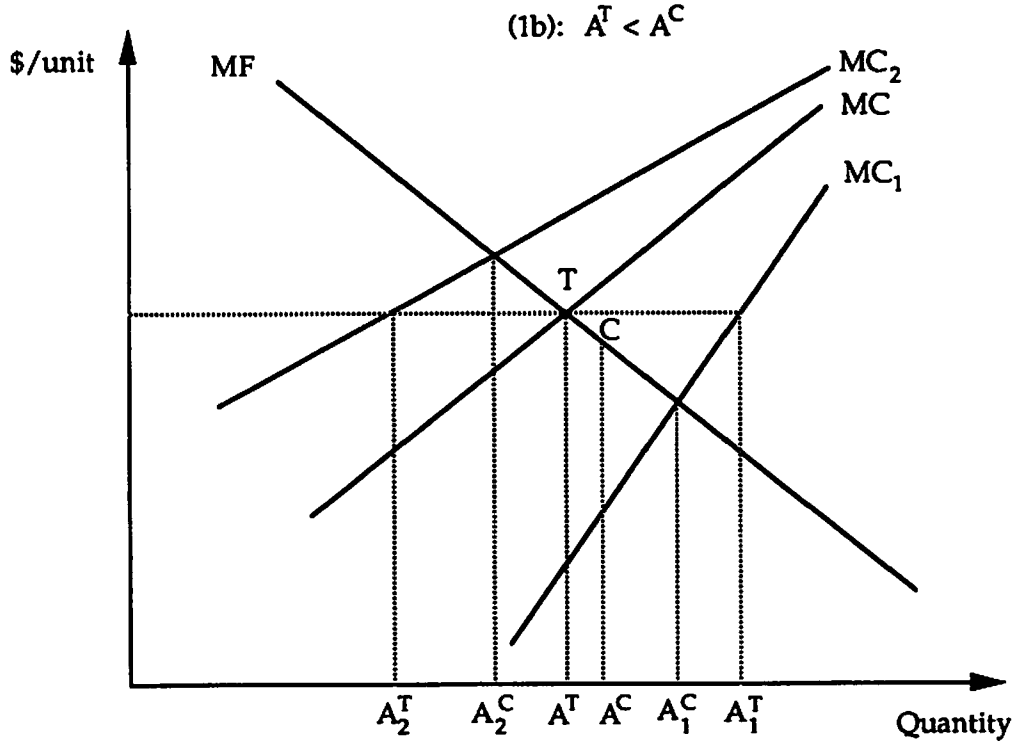
[9]Total control can be obtained by multiplying by the total number of objects, which is presumed to be the same under both trading and command-and-control.

Figure 1: Comparison of Command-and-Control and Trading Solutions

(1a): $A^T > A^C$



(1b): $A^T < A^C$



C on the MF curve, which represents the midpoint of the segment of this curve between the two marginal cost curves.

The solution to the trading problem is more subtle. Let MC denote the weighted aggregate marginal cost curve. The trading equilibrium is given by the intersection of MC and MF, which is denoted by point T. The controls under trading are determined by setting MC_1 and MC_2 equal to the value of MF at A^T, and noting the corresponding values of A_1^T and A_2^T.

While this solution to the trading problem characterizes the different control levels, it does not explicitly show how the individual marginal expected penalty functions are selected for each class, nor does it identify the choice of performance standards. The expected penalty functions are selected implicitly through the choice of S_i for each class. If $S_i >(<) S$, MF shifts to the left (right) by the absolute value of the difference between S_i and S. The individual standards, S_1 and S_2, are not shown explicitly, but are easily identified by applying Lemma 1: $S_1 = S - (A_1^T - A^T)$ and $S_2 = S + (A^T - A_2^T)$.[10]

Having constructed the equilibrium, it is straightforward to compare the outcomes under trading and command-and-control. From Figure 1a, it is clear that $A^T > A^C$. Note, however, that this result is reversed in Figure 1b. Moreover, the only change in Figure 1b is that the MC_1 curve has been rotated around the intersection with the MF curve so that its slope is now steeper than MC_2.[11]

The intuition for the result follows from examining the effect of reallocating the standard across classes to a change in total control. Again, consider Figure 1a. In this particular case, the standard for class 2 increases by

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[10]It is straightforward to check the first order conditions. $MC_i = F'(-A_i - S_i)$ (for i = 1, 2) by construction, so (5a) is satisfied. Letting the equilibrium marginal cost equal $-\lambda$, (5b) is satisfied. To show (5c) is satisfied, first note that A^T represents the midpoint between A_1^T and A_2^T (from the construction of C'). Thus, $A_1^T - A^T = A^T - A_2^T$. Direct substitution then shows that (5c) is satisfied.

[11]If MC_1 had been rotated so that its slope just equaled MC_2, then average control for the two cases would have been the same, i.e., A^T would equal A^C.

TX, while the standard for class 1 decreases by TY, which equals TX. The equal and opposite change in the standard is required in order to ensure that the weighted standard just equals S. The change in the standard results in a parallel shift inward of the MF curve for Class 2 and outward for Class 1 so that the curve for Class 2 now passes through point X, yielding A_2^T; and the curve for Class 1 passes through Y, yielding A_1^T. The key to understanding the general result is to note two points: first, that the impact of a parallel shift in the MF function on control will depend on the shape of the particular MC function; and second, that the change in control is more pronounced for a flatter MC function than a steeper MC function. Thus, because MC_1 is flatter than MC_2, the increase in control under trading for class 1 exceeds the decrease for class 2.

Another way of seeing the result is to compare the marginal cost and expected penalty functions at the command-and-control solution. In Figure (1a), the marginal cost is less than the marginal expected penalty at A^C. Thus, firms are induced to move to the higher control levels when trading is permitted, so as to allow C'= F'. Thus, in general, the relative position of A^C and A^T is determined by the relative position of C' and F' at A^C. If $C'(A^C)<(=)(>)F'(-A^C-S)$, then $A^T > (=)(<) A^C$.

It is instructive to link this analysis to the textbook case in which a reallocation of the standard has no impact on total control. If the MF curve were vertical, then the solutions under T and C. Of course, this case is likely to be the exception rather than the rule when enforcement is imperfect.

The ideas in Figure 1 can be extended with the help of a concept that links the position and slope of the marginal cost curves.

<u>Definition</u>: The marginal cost curves are said to be <u>positively (negatively) ordered</u> at the trading equilibrium, $A^T = (A_1^T,...,A_n^T)$, provided that $A_i \neq A_j$ for all $i \neq j$, and $A_i^T > A_j^T$ iff $C_i'' < (>) C_j''$.

Figure (1a) corresponds to a case where the marginal cost curves are positively ordered while Figure (1b) corresponds to a case in which the curves are negatively ordered.

This definition is useful in developing the following theorem:

Theorem 1: Assume that T and C have interior solutions with controls A^T and A^C. If there are n linear marginal cost schedules ($n \geq 2$) that are positively (negatively) ordered and the marginal expected penalty curve is concave (convex), then $A^T > (<) A^C$.

Proof: See appendix.

This result provides rather restrictive conditions on the cost functions in signing the relative amounts of average and total control in the two cases. At the same time it suggests that the MF curve is not particularly critical for the result, provided that the relative position and slopes of the MC curves do not change at A^T and A^C. In terms of Figure 1a, suppose MF were shifted to the right or the left by a small amount, it is clear that the new solutions to the problem would also satisfy the theorem. This is not to say that the position of the expected penalty function does not matter. For example, if there were two linear marginal cost curves that intersected in the positive orthant, then the location of the MF curve would be critical for determining how to apply the theorem.

Note also that the theorem states that the situation depicted in Figure 1a can be generalized to a case in which MF is concave. Similarly, Figure 1b can be generalized to the case in which MF is convex.

The theorem also can be generalized to the case where output varies with abatement. This involves redefining the direct cost function to be the $\Sigma C_i(A_i)$, where $C_i(A_i)$ now is interpreted as the total cost of control for class i after taking account of output effects. The same qualitative results will obtain for this modification provided the cost functions are convex.

The fact that control can increase, decrease or remain the same in moving from the command-and-control case to trading has some interesting implications for costs. In terms of total costs, which includes direct costs plus penalties, outlays under the command-and-control problem will be greater

than or equal to the trading problem. This is because any solution to the command-and-control problem is also feasible for the trading problem by setting $S_i = S$.

The relationship of direct control expenditures (those involving C, but not F) is more subtle, and differs from the classical result that direct costs under trading never exceed those incurred under command-and-control. In this model, the classical result will obtain provided that control under T does not exceed that under C, and T has an interior solution.

If control increases, the impact on direct control costs can be positive, negative or neutral. The extreme cases are illustrated in Figure 2. Note that $A^T > A^C$ in both Figure 2a and Figure 2b. The magnitude of the cost change is given by the relevant areas under the marginal cost curves in moving from C to T. In Figure 2b, the cost increase in moving from C to T, noted by the shaded region with a "+" sign, is less than the cost decrease, noted by the shaded region with a "-" sign. Thus, direct costs decrease on net. Just the opposite case is illustrated in Figure 2b. Note also that the only difference between Figures 2a and 2b is that the MC_2 is rotated so that it has a steeper slope. As the slope of MC_2 in the figure increases toward infinity, the cost savings trapezoid approaches zero area and it must be the case that direct control costs increase in moving from C to T. This illustration also suggests the case of no change in direct costs can be constructed by rotating the MC_2 curve in Figure 2b back towards the MC_2 in Figure 2a until the two trapezoidal areas are equal.

The preceding theorem assumed that there was only one expected penalty function. It is possible to develop a result for n expected penalty functions, but the conditions are rather restrictive. Instead, we consider a simpler problem in which there are n identical cost functions and n different expected penalty functions. Identical cost functions imply that each class uses the same level of control, denoted as "A". This problem will be useful in analyzing some subtle enforcement issues. First, the concept of ordering needs to be defined for the expected penalty functions.

Figure 2: The Effect of Trading on Direct Control Costs

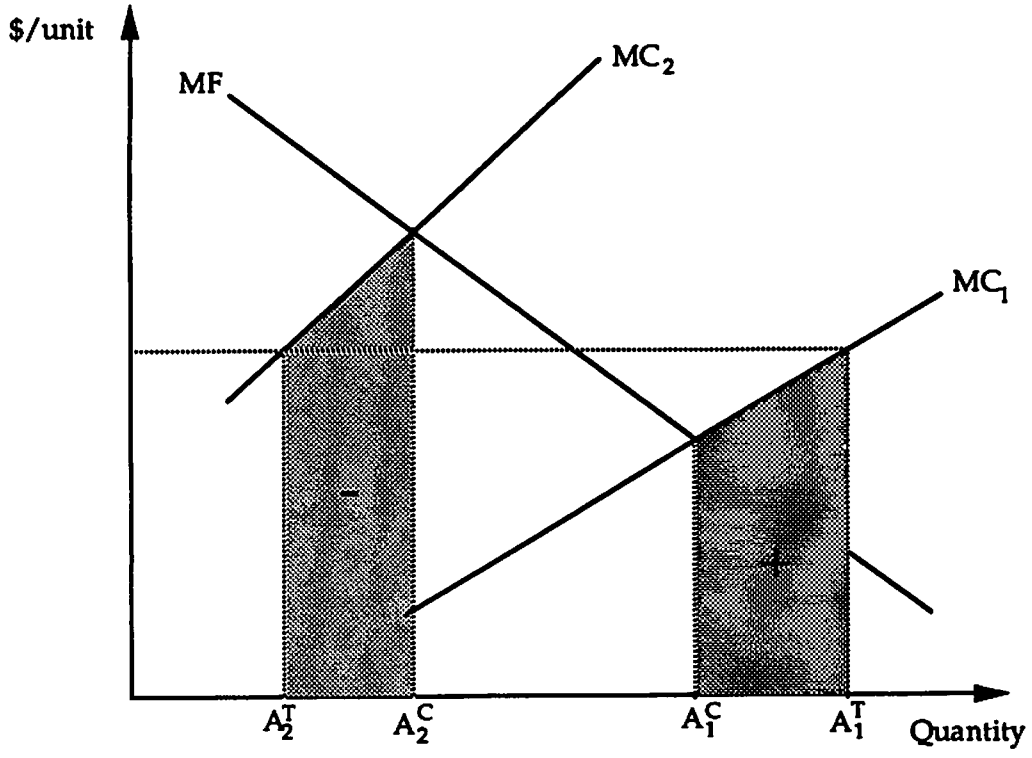

Figure 2a: Direct Costs Decrease

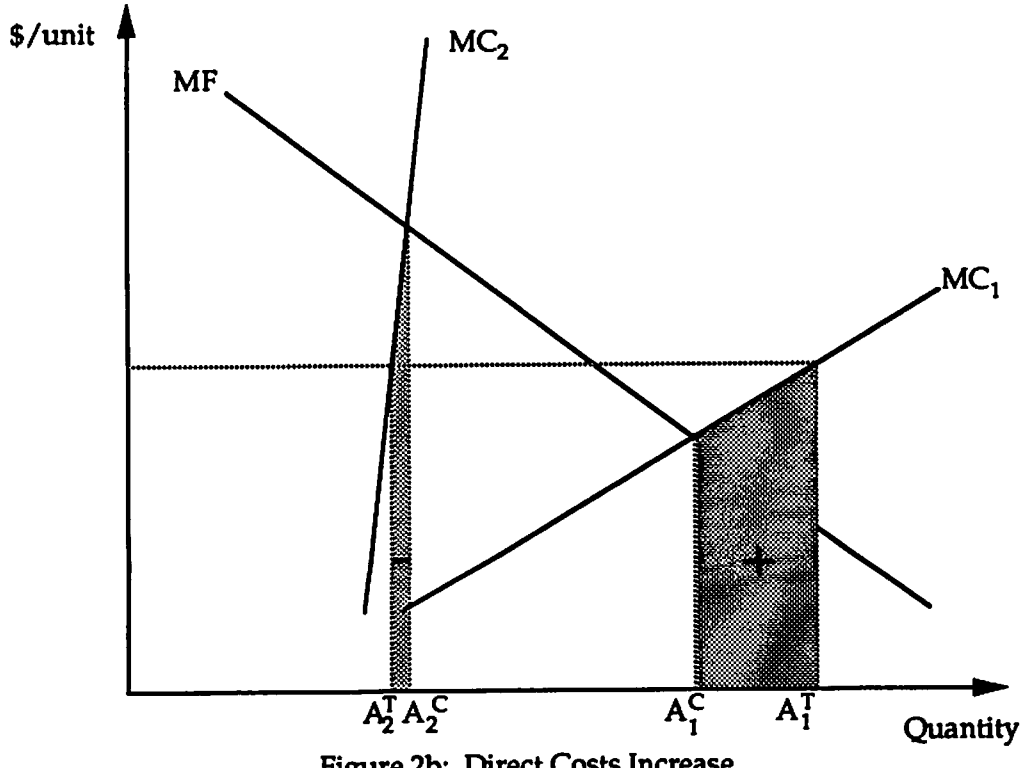

Figure 2b: Direct Costs Increase

Definition: The marginal expected penalty curves are said to be positively (negatively) ordered at the trading equilibrium, A^T, provided that $S_i \neq S_j$ for all $i \neq j$ and $A - (S_i - S) > A - (S_j - S)$ iff $F_i'' > (<) F_j''$.

The value $A - (S_i - S)$ represents the level of control at the intersection of F_i' and the equilibrium price of a property right when this function is evaluated at the standard, S. Thus, in the case of a positive ordering, the marginal expected penalty curves are ordered from right to left at the equilibrium price with the most steeply sloped curve on the left. The slopes are reversed in the case of a negative ordering. This ordering concept yields the following theorem:

Theorem 2: Assume that T and C have interior solutions with controls A^T and A^C. If there are n linear marginal expected penalty schedules ($n \geq 2$) that are positively (negatively) ordered, then $A^T > (<) A^C$.

Proof: See appendix.

As noted above, the general case of different fine and cost functions is much more difficult to analyze. For the special case in which marginal cost functions are parallel to each other and marginal expected penalty functions are parallel to each other, it is possible to show that $A^T = A^C$. What is interesting about this result is that no particular ordering needs to be imposed on the schedules. The intuition behind this result is that any shift in a particular marginal expected penalty schedule resulting from a reallocation of the standard will need to be matched by an equal and opposite shift of other expected penalty schedules. The impact on the total control from these shifts will cancel out because both sets of curves are parallel.[12]

Finally, while not the focus of this paper, it is useful to explore how the comparison of command-and-control might compare with various taxation systems. In keeping with the spirit of the literature on emission taxes, we

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[12]Indeed, for this same reason, it is possible to show that n parallel marginal penalty schedules will produce the same solution under trading as a weighted horizontal aggregation of these schedules. See appendix for the formal proofs of these results.

consider two possibilities. First a tax is placed directly on emissions. A tax on emissions yields the unambiguous result that emissions decrease. The reason is that a tax on emissions results in a vertical shift up in the marginal expected penalty function by the amount of the tax, thus increasing the level of control. Suppose, instead of an emissions tax, a tax were placed directly on the vector of performance standards selected by the firm. In this case, it is possible to show the standard duality result between taxes and transferable property rights. In particular, if the tax is selected so that it reflects the marginal value of relaxing the standard at a specified level of control, it yields the same outcome as a system of transferable property rights.[13]

To summarize, this section has shown that the move to a transferable property rights scheme from a command-and-control approach will not be more costly, but it could increase direct control costs (if control increases) or, alternatively, it could decrease the amount of control. The presumed superiority of transferable rights systems is usually based on the argument that they represent a less expensive mechanism for achieving a given objective.[14] When the objective can no longer be specified with certainty because of imperfect monitoring and enforcement, this presumed superiority is just that, a presumption.

3. Interpretation of the Model

Like most formal economic models, this one gives rise to a variety of interpretations. This section focuses on three issues: defining the precise nature of the dominance result; exploring how changes in the structure of enforcement could affect the dominance relation; and finally, analyzing whether markets in transferable property rights intrinsically create more loopholes than a command-and-control regime.

A. The Dominance Result

[13]See appendix for the proof.

[14]The implicit definition of superiority has obvious limitations in that it need not correspond to improvements in economic welfare. In the interest of simplicity, we ignore such problems.

The standard argument for the "dominance" of transferable property rights over command-and-control is that transferable property rights can achieve the same aggregate outcome at the same or a lower cost. The preceding theorems show this dominance result need not obtain in the case of imperfect monitoring and enforcement. In particular, the result shows that by choosing the same performance standard, S, the agency cannot determine a priori (except in special cases) how the aggregate control under trading will compare with the aggregate control under command-and-control. Thus, the result highlights the difficulty of designing any new economic system that will reach a certain goal when there is slippage between the stated goal and what is actually realized. In this sense, the result merely reinforces the notion of the tyranny of the status quo.

It is important to recognize that the result does not call into question the comparative productive efficiency resulting from instituting a system of transferable property rights. This follows immediately from equating marginal costs across all classes and the assumption that the penalty structures are similar across the two institutions. Only in cases where there are substantial differences in administrative costs across systems would this result come into question.

If transferable property rights are at least as efficient, then it is natural to conjecture that one could define a system of such rights that dominated command-and-control. In principal, with perfect information, this could be done. For example, suppose the amount of control under a command-and-control regime could be measured accurately. Then, it is possible to show there exists a performance standard (generally not equal to the standard used under command-and-control), such that the control achieved with that standard under a transferable property rights regime would equal the control achieved under an existing command-and-control regime. Of course, the problem is that the regulator does not have the information to pick such a standard in this simple economic framework used here. Moreover, we believe this characteristic is not simply an artifact of our model, but rather is shared by a host of real-world environmental problems.

Since transferable property rights are generally more efficient, it is reasonable to ask why we have chosen to focus on this narrow definition of dominance, which we candidly admit may have little to do with social welfare (assuming we could define the latter). We focus on the question of dominance for two reasons: First, because the conventional wisdom in the literature suggests that such a result obtains; and second, because in the real world, there are many classes of problems for which markets would be viewed as attractive if a dominance result held (see, e.g., Stavins, 1991).

B. Changing the Penalty Function

For analytical purposes, marginal expected penalty functions have been held constant across the trading and command-and-control institutions. This was done in the interest of comparing the institutions under comparable conditions. In reality, of course, the policy maker can exert some control over the expected penalty function. Thus it is interesting to inquire what would happen with a change in the expected penalty function.

We shall focus on the case of trading because this is probably the more relevant case for policy design.[15] A casual inspection of Figure 1a and Figure 1b suggest that an outward shift in the MF curve increases average control under trading, A^T. Because $C'(A^T)$ also increases with an outward shift in MF, the equilibrium price of a permit will also increase. Loosely speaking, this result says that more vigorous monitoring and enforcement of property rights would tend to raise their price because the asset becomes more valuable. In the case of pollution, for example, this says that increasing the marginal expected penalty for polluting would increase abatement, and thus increase the value of the transferable property rights associated with this pollution. While shifting the expected penalty function in a parallel fashion would increase the amount of control under trading, it would also increase direct control costs, and thus make the system less attractive. An alternative is to rotate the expected penalty function so that, in the limit, it is vertical. In

[15]The case of command-and-control is straightforward. From a casual inspection of Figure 1a and Figure 1b it is clear that an outward shift of the -MF curve will increase the amount of control.

terms of Figure 1a, this would mean that MF is rotated around C until it is vertical. In this case, it trivially follows that $A^T = A^C$. While attractive as a potential theoretical solution, it may not be technically feasible to have what is essentially perfect enforcement, and even if it were feasible, it could be quite costly.

The policy implication of this result is important. To the extent that regulators are under pressure to achieve higher levels of control under trading, as is the case with environmental problems, they can do so by shifting the MF function, for example by decreasing the average performance standard. At the same time, however, policy makers will, in general, not be able to guarantee that A^T with a modified expected penalty function will exceed A^C under the original expected penalty function. Only if policy makers know a great deal about the cost functions, or they are willing to impose draconian penalties at the margin, will they then be able to be confident that $A^T > A^C$. Except for the case when MF becomes vertical at C, there is no guarantee that the dominance result will obtain. Moreover, even in this case, the potential increase in administrative costs incurred by the government could far exceed the potential savings in direct control costs.

C. Do Markets Necessarily Create Loopholes?

Opponents of markets in transferable property rights often claim that they create loopholes that do not exist under command-and-control regulation. The precise nature of this claim can be fruitfully analyzed within the context of Theorems 1 and 2.

With the same expected penalty function, Theorem 1 clearly states there are situations in which markets will lead to higher control and there are situations in which markets lead to lower aggregate control. If lower control is interpreted as resulting from a "loophole" then markets, indeed, can create loopholes; however, Theorem 1 makes it clear that such markets need not create loopholes.

Another perspective on the problem of loopholes is provided by Theorem 2, which considers the more realistic case of multiple expected

penalty functions. For the simple case in which cost functions are identical, this theorem shows that loopholes may or may not result, even in the case of multiple expected penalty functions. Moreover, in general, the result will depend on the shape of the expected penalty functions and the cost functions.

Both theorems assume that the expected penalty and cost functions remain the same across institutions. Of course, it is probably unreasonable to assume that expected penalty functions will necessarily stay the same across institutions, even if regulators so desired. The reason is that the institutions are fundamentally different in many respects. For example, under a transferable property rights market, firms are given more flexibility in meeting a given objective. There is less of a need for lawyers to be engaged in rule writing for regulating particular classes of objects. Yet, there may be greater potential for counterfeiters to enter the system and profit, since there is more flexibility in the system and property rights that are generated illegally could be traded for money. As a result, transferable property rights systems and command-and-control systems may require fundamentally different monitoring and enforcement technologies to yield the same expected penalty functions.

Our belief is that the concern about loopholes is really a concern that the expected penalty functions, contrary to our assumption, will not remain constant across institutions. But as shown above, the expected penalty functions would have to change in specific ways in order to result in loopholes. At least in the environmental arena, the evidence suggests that the concern with loopholes is overstated (Hahn, 1989).[16]

4. Implementing the Model

If cost functions and expected penalty functions were readily estimated, the model would be straightforward to implement. This happy state of affairs

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[16]For example, in moving to a market-based system for controlling acid rain in the U.S., firms participating in the market were required to install continuous emissions monitors, which gives the agency an enhanced monitoring capability in comparison with command-and-control. This requirement was in addition to the targeted reduction of almost one-half of current sulfur dioxide emissions by the year 2000.

tends to be the exception rather than the rule in environmental applications. While it is frequently possible to estimate control cost functions for reducing pollution, rarely is it possible to measure directly the expected penalty function. Firms interested in maintaining a reputation for being law-abiding and producing a quality product would not likely equate their marginal cost of controlling pollution with the marginal expected monetary penalty.

Fortunately, the expected penalty function can be inferred by using the cost functions along with the first order conditions. As an example, we considered the case of "vehicle emissions trading". Vehicle emissions trading refers to any rule that allows manufacturers to implicitly or explicitly trade off emissions among different parts of a vehicle or different vehicles so long as the overall level of emissions reductions are achieved. Currently, there is a narrowly defined program that allows trading for nitrogen oxides and particulate emissions from engines used in heavy duty trucks.[17]

The Administration's Clean Air Act bill called for expanding the role of vehicle emissions trading to light duty vehicles. The idea was to allow vehicle manufacturers greater flexibility in meeting an overall emissions standard by allowing some vehicles to emit above this standard and others to emit below the standard. This proposal was met with fierce resistance in Congress. Congressman Henry Waxman, Chairman of the House Subcommittee on Health and the Environment, argued that "the bill's averaging provision actually relaxes today's existing requirements. Averaging will allow auto pollution levels to increase, and will eviscerate the warranty and recall sections of the current law."[18]

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[17]California is also considering an averaging program that would allow limited emissions trading within narrowly defined classes for low-emitting vehicles.

[18]Statement by Congressman Henry Waxman, U.S. House of Representatives, Subcommittee on Health and the Environment (1989, p. 4). In pre-hearing handouts, members of the Health and Environment Subcommittee received a background paper from Waxman's staff to help shape their views on averaging. They were given a hypothetical example based on the Cubs and the Dodgers. The Cubs lineup had a batting average of .300, while the Dodgers average was .322, and all members in the lineup were hitting above .300. The recipients of the handout were asked, "Which is the better hitting team?". The not-so-subtle implication was that averaging (i.e., trading) always leads to a worse outcome than command-and-control.

Waxman's argument rested on the view that the margin of safety that manufacturer's build into their design to pass emissions tests would decline upon introducing a market. As Theorem 1 suggests, this could be the case, but it is not a foregone conclusion. To examine this proposition, data were assembled on the control costs associated with reducing carbon monoxide for large and small cars.

Federal regulations for carbon monoxide limit emissions to 3.4 grams/mile for all cars. An alternative system would be to allow manufacturers to vary the standard across engine families, so long as the weighted average of the designated standards is less than or equal to the current standard. Under such a scheme, auto-makers would be required to announce the performance standards of each engine family in advance of Federal tests used to determine whether the standards would be met.[19]

To compare these two institutions, control cost functions are estimated using data from two sources -- one that describes pollution control costs and a second that estimates effectiveness.[20] The cost data provide information on the marginal cost of different components for pollution control on different vehicles. While the data are supplied for a large number of vehicles, we consider two engine families that are representative of "large" and "small" cars. These data are then linked to data on emissions reductions associated with various control devices. The emissions estimates are based on the Environmental Protection Agency's Federal Test Procedure for motor vehicles. The calculation yields two marginal cost of abatement schedules. The expected penalty function is then inferred by assuming that firms are operating on their marginal cost of abatement schedules for each vehicle class at the designated performance standard. The estimation procedure yields two points on the marginal expected penalty function, which are then used to construct a linear marginal expected penalty function.

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[19]For a description of the testing procedures see White (1982).

[20]Cost data included information on the following technologies: exhaust gas recirculation systems, air injection units, catalytic converters, ignition systems, fuel metering devices, evaporative emissions control technology, electronic control units, closed-loop sensors, and diagnostic systems. The data used to generate these results are available from Mr. Axtell.

Given the uncertainties in the estimation procedure, which reflects the state-of-the-art, a series of sensitivity analyses were run varying parameters in the cost functions for the large and small vehicle classes. Two conclusions emerged from this application regarding environmental outcomes. First, on the basis of the data, one could not reject the hypothesis that emissions remained the same in moving from a command-and-control system to a transferable property rights system (and vice versa). Second, the results were quite sensitive to the level of uncontrolled emissions for each vehicle class, which help to define the intercepts of the marginal cost schedules. Finally, we suspect that for most real-world applications, it would be quite difficult to say with any degree of precision how the transition from one institution to another might affect environmental quality unless the marginal expected penalty function is approximately vertical under the new institution.

5. Conclusions and Areas for Future Research

This paper has tried to advance both our theoretical and empirical understanding of markets in transferable property rights. The theoretical model demonstrated the precise nature of difficulties in designing transferable property rights schemes that dominate command-and-control regimes in a static environment. In particular, it showed that the "safety margin" need not be smaller with the introduction of a market scheme, contrary to conventional wisdom. The empirical discussion illustrated one approach to implementing the theoretical model, and highlighted the difficulties in making unambiguous statements about environmental outcomes given the uncertainties in the data. Clearly, more work is needed.

Three theoretical extensions within the basic model would be useful. One would consider more sophisticated expected penalty functions. A second would examine how uncertainty is likely to affect the results. For example, it could be the case that, given uncertainties in the cost functions and expected penalty functions, it may be impossible to devise systems that can attract the needed political support, because such systems cannot guarantee specific outcomes (such as halving automobile pollution) with sufficient confidence.

Finally, an issue of great importance that has not received adequate attention is how different institutions, such as the ones examined here, are likely to affect the path of technological progress.

On the empirical side, the primary problem is how to estimate expected penalty functions that would give some insight into the practical relevance of the theory. To put it politely, we know very little about the shape of such functions in actual practice.

Beyond questions of basic research, there is a pressing need to examine problems in market design and implementation (Noll, 1982). One of the most exciting potential "growth areas" is in the application of markets and other incentive-based approaches to environmental problems (Stavins, 1988). Politicians are increasingly willing to consider market-based approaches as a legitimate alternative in addressing policy concerns. A key challenge for the academic community is to enrich the menu of useful policy tools available to elected officials and bureaucrats.

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Appendix

This appendix provides proofs for the basic results of the paper and also provides some extensions that are briefly discussed in the text.

Lemma 3: An interior solution to the trading problem is completely characterized by the intersection of the weighted marginal cost curve, $C'(A)$, and the marginal expected penalty function, $F'(-A-S)$.

Proof: The proof to the lemma is constructed in two parts. First, an interior solution is shown to satisfy the point where $C' = F'$. Then the point where $C' = F'$ is used to generate a solution satisfying the first order conditions (5a) - (5c). Suppose an interior solution to the trading problem exists, and let A_i^*, S_i^*, $i = 1,\ldots,n$, represent the solution. Let $A^* = \Sigma n_i A_i^*$. From the definition of C', it follows $C'(A^*) = C_i'(A_i^*)$ for all i. Thus, it suffices to show $F'(-A^*-S) = C_i'(A_i^*)$. From (5a), $C_i'(A_i^*) = F'(-A_i^* - S_i^*)$. Note the desired result is true if $A^* + S = A_i^* + S_i^*$.

$$
\begin{aligned}
A^* + S &= \Sigma n_i A_i^* + S, \text{ from the definition of } A^* \\
&= \Sigma n_i (A_i^* + S_i^*), \text{ from (5c) and factoring,} \\
&= A_i^* + S_i^*, \text{ from Lemma 2 } (A_i^* + S_i^* \text{ equal), and } \Sigma n_i = 1,
\end{aligned}
$$

which proves the first part of the lemma.

To show the second part, assume that A^o is the A that solves $C'(A) = F'(-A-S)$. Let $A_i^o = C_i'^{-1}(C') = A_i^*$, so that $A^o = \Sigma n_i A_i^o$. Let $S_i^* = A^o + S - A_i^o$. The problem is to show that this A_i^* and S_i^* satisfy conditions (5a) through (5c). The first condition says $C_i'(A_i^*) = F'(-A_i^* - S_i^*)$, for $i = 1,\ldots,n$. Note that $C'(A^o) = C_i'(A_i^*)$ (by construction) and that $F'(-A_i^* - S_i^*) = F'(-A^o - S)$, since $A_i^* + S_i^* = A^o + S$ by construction. Substitution of these two equalities yields the desired result. Letting $F'(-A^o - S) = -\lambda$ provides a solution to condition (5b), since $A_i^* + S_i^* = A^o + S$ by construction. To show that (5c) is satisfied, note that

$$
\begin{aligned}
\Sigma n_i S_i^* &= \Sigma n_i (A^o + S - A_i^o), \text{ from construction,} \\
&= \Sigma n_i S + \Sigma n_i A^o - \Sigma n_i A_i^o, \text{ by expanding terms,} \\
&= S + A^o - \Sigma n_i A_i^o, \text{ since } \Sigma n_i = 1, \\
&= S, \text{ from definition of } A^o,
\end{aligned}
$$

which completes the proof of the lemma.

Theorem 1: Assume that T and C have interior solutions with controls A^T and A^C. If there are n linear marginal cost schedules ($n \geq 2$) that are positively (negatively) ordered and the marginal expected penalty curve is concave (convex), then $A^T > (<) A^C$.

Proof: The proof proceeds in two stages. First, the case of a linear MF function is proved. Then, the result is extended to a more general marginal expected penalty function.

Without loss of generality, order the A_i^T so that $A_1^T > A_2^T > \ldots > A_n^T$. From Lemma 1, it immediately follows that $S_1^* < S_2^* < \ldots < S_n^*$ where S_i^* are the optimal standards in the trading solution. Define

(A1) $C_i'(A_i) = a_i + b_i A_i$, $i=1,\ldots,n$.

Recall that, by assumption, $C_i''>0$, which implies $b_i>0$ for all i.

If the marginal cost curves are positively (negatively) ordered, then $b_i <(>) b_j$ iff $A_i^T > A_j^T$. This follows from the ordering of A^T and the definition of ordered marginal cost curves.

The slopes of the marginal cost curves can now be linked to the optimal standards as follows:

If marginal cost curves are positively (negatively) ordered, then

(A2) $b_i <(>) b_j$ iff $S_i^* < S_j^*$.

This result is a generalization of the illustration in Figure 1 and follows immediately from Lemma 1 and the definition of ordering. Define

(A3) $F' = c + d\Delta$,

where Δ is the negative of the sum of the control plus standard, but varies depending on the problem (see (2), (5a) and (5b)). Note that $F''>0$ implies that $d>0$.

Substituting (A1) and (A3) into (2) and (5a) yields:

(A4) $A_i^C = (-dS + c - a_i) / (b_i + d)$

and

(A5) $A_i^T = (-dS_i^* + c - a_i) / (b_i + d)$.

The problem is to compare $\Sigma n_i A_i^T$ with $\Sigma n_i A_i^C$.

$\Sigma n_i A_i^T > (<) \Sigma n_i A_i^C$ iff

$\Sigma(-dn_i S_i^*)/(b_i + d) > (<) \Sigma(-dn_i S)/(b_i + d)$ iff

(A6) $\Sigma(n_i(S_i^*-S))/(b_i + d) < (>) 0.$

The first equivalence follows from substitution of (A4) and (A5) after subtracting the term $(c - a_i) / (b_i + d)$, which is common to both. The second equivalence follows from dividing by -d, transposing and factoring.

Consider the case where the marginal cost curves are positively ordered, which means that it suffices to show $\Sigma(n_i(S_i^*-S))/(b_i + d) < 0$. The essence of the proof will be to petition the terms of (A6) within the summation sign into two groups, those that are nonnegative, and those that are negative. Before doing this, note that the sum of the terms in the numerator is zero. That is:

$$\begin{aligned}\Sigma n_i(S_i^*-S) &= \Sigma n_i S_i^* - \Sigma n_i S \\ &= S - S\Sigma n_i \\ &= S - S \\ &= 0.\end{aligned}$$

Now, let $\delta_i = S_i^*-S$. From the preceding equality, it follows that

(A7) $$\left|\sum_{i:\delta_i<0} n_i\delta_i\right| = \left|\sum_{i:\delta_i\geq 0} n_i\delta_i\right|.$$

It follows from (A1) that the negative δ_i's will be associated with the low b_i's and the positive δ_i's will be associated with the high b_i's.

Define j as the highest index for which $\delta_j<0$. Then

$$\left|\sum_{i=1}^{j} n_i\delta_i/(b_i + d)\right| \geq \left|\sum_{i=1}^{j} n_i\delta_i/(b_j + d)\right|$$

$$> \quad \left| \sum_{i=j+1}^{n} n_i\delta_i/(b_{j+1} + d) \right|$$

$$\geq \quad \left| \sum_{i=j+1}^{n} n_i\delta_i/(b_{i+1} + d) \right|,$$

where the first inequality is true because $b_j \geq b_i$ so each term in the right hand side sum is lower in absolute value, but the same sign as the left hand side. The second inequality follows from (A7), given the denominator, $(b_j + d)$ is positive and less than $(b_{j+1} + d)$. Finally, the last inequality is true because the terms in the denominator are replaced by terms that are positive, but lower or the same in value.

By transitivity, the preceding results yield:

$$\left| \sum_{i=1}^{j} n_i\delta_i/(b_i + d) \right| > \left| \sum_{i=j+1}^{n} n_i\delta_i/(b_{i+1} + d) \right|.$$

Noting that the terms within the summation on the left hand side are negative and the terms on the right hand side are nonnegative gives:

$\Sigma(n_i(S_i-S))/(b_i + d) < 0$, which is the desired result.

The proof for the case in which the marginal cost curves are negatively ordered is virtually identical. Just define j as the highest index for which $\delta_j \geq 0$, and reverse the inequality signs, and the proof is complete.

This idea can be extended to the case in which MF is concave (convex). Again, the proofs are quite similar, so only the proof for MF concave will be presented in detail. First, construct the C'(A) line as above and define T as the intersection of this line with MF. Draw the tangent to MF at T and call this MF*. Introduce the following superscripts to characterize solutions to the problem: T represents the trading solution to the original problem and T* represents the trading solution to the linearized problem. Correspondingly, C represents the command-and-control solution to the original problem and C* represents the solution to the linearized problem.

It suffices to show that $\Sigma n_i A_i^T = \Sigma n_i A_i^{T*} > \Sigma n_i A_i^{C*} \geq \Sigma n_i A_i^C$

since this implies $\Sigma n_i A_i^T > \Sigma n_i A_i^C$ by transitivity. The first equality is demonstrated by showing that A_i^T, S_i^T, $i = 1,\ldots,n$ and λ represent a solution to the linearized problem. The only difference in the first order conditions, (5a) through (5c) to the two problems is the slope of the marginal expected penalty function. By construction, however, the slopes are the same at T. Applying Lemma 1, it immediately follows that the solution to the trading problem also will satisfy the conditions for an interior minimum to the linearized trading problem, and thus is the solution.

The first inequality follows immediately from an application of the linear version of the theorem for the case in which there are linear positively ordered marginal costs curves. The final inequality follows from the fact that $A_i^{C*} \geq A_i^C$, given the construction of the tangent line and the positive slopes of the marginal cost curves.

The proof for the case of linear negatively ordered marginal cost curves is the same except the signs for the inequalities are reversed.

Theorem 2: Assume that T and C have interior solutions with controls A^T and A^C. If there are n linear marginal expected penalty schedules ($n \geq 2$) that are positively (negatively) ordered, then $A^T > (<) A^C$.

<u>Proof</u>: The proof of this theorem follows directly from the proof of the linear case for Theorem 1 with a modification to reflect that the expected penalty functions can differ across classes. This yields the following expressions for A_i^C and A_i^T, which is equal to A since the cost functions are identical.

(A8) $A_i^C = (-d_i S + c_i - a) / (b + d_i)$

and

(A9) $A_i^T = (-d_i S_i + c_i - a) / (b + d_i)$.

Equation (A6) then becomes:

(A10) $\Sigma(n_i(S_i^* - S))[d_i/(b + d_i)] < (>) 0$.

By reasoning directly analogous to the linear case for Theorem 1, it suffices to show that $d_i/(b + d_i)$ is decreasing in i for the positive ordering and increasing in i for the negative ordering. The definition of positive (negative) ordering of the expected penalty functions implies $d_i >(<) d_j$ iff $S_i^* < S_j^*$. This immediately yields the desired result.

Lemma A1: Assume that T and C have interior solutions. If there are n parallel linear marginal cost functions with a positive slope n parallel marginal expected penalty functions with positive slope, then $A^T = A^C$.

Proof: The proof of this theorem follows directly from the definitions. Parallel marginal cost functions and parallel marginal expected penalty functions yield the following controls:

(A11) $A_i^C = (-dS + c - a) / (b + d)$

and

(A12) $A_i^T = (-dS_i^* + c - a) / (b + d)$, where the "*" denotes part of the solution.

It suffices to show that $\Sigma\, n_i(A_i^T - A_i^C) = 0$.

$$\begin{aligned} \Sigma\, n_i(A_i^T - A_i^C) &= \Sigma\, n_i(S_i^* - S)\,(d/(b+d)) \\ &= (d/(b+d))\, \Sigma\, n_i(S_i^* - S) \\ &= 0, \end{aligned}$$

where the first equality follows from substituting (A11) and (A12) and canceling like terms; the second equality follows from taking a constant out of the sum, and the third equality follows from noting the sum is zero. (See the earlier derivation following (A6)).

Lemma A2: If there are n parallel marginal expected penalty functions, the solution under trading can be replicated by a single marginal expected penalty function, which represents the weighted average of the original marginal expected penalty functions.

Proof: The proof is sketched here. Suppose the original n marginal expected penalty functions have the following form:

(A13) $F_i'(-A_i - S_i) = c_i + d\,(-A_i - S_i)$ for $i = 1,\ldots,n$.

Define a new marginal expected penalty function, G, that has the following form:

(A14) $G'(-A_i - S_i) = \Sigma\, n_i c_i + d\,(-A_i - S_i)$ for $i = 1,\ldots,n$.

Suppose $A_1^*,\ldots,A_n^*, S_1^*,\ldots,S_n^*, \lambda^*$ solve the trading problem with n expected penalty functions. We will show that this solution will satisfy the original problem iff $A_1^*,\ldots,A_n^*, S_1^{**},\ldots,S_n^{**}, \lambda^*$ solve the modified trading problem, where $S_i^* = S_i^{**} + (c_i - \Sigma\, n_i c_i)/d$.

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The Center for Science and International Affairs (CSIA) is dedicated to advancing the understanding and resolution of complex public policy problems of international scope through research at the intersection of the natural and social sciences. The number of such problems is growing, and the Center's research agenda currently encompasses international security affairs, environment and natural resources policy, and the role of science and technology in shaping the international economy and other global concerns. Each year a multinational group of predoctoral and postdoctoral scholars drawn from the social and natural sciences is in residence at the Center. More than 50 Harvard faculty members and about 70 non-resident affiliates are also involved in Center activities.

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The Program involves faculty and senior researchers from the Kennedy School, as well as other professional schools at the university. Workshops, executive sessions, summer internships for students and the Discussion Paper series are also a part of the ENRP program.

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subject to: $A_i \geq 0$ and $S_i \geq 0$ for $i = 1,\ldots,n$,

where "t" is the tax.

The first order conditions for an interior minimum are

(A22) $C_i'(A_i) - F_i'(-A_i-S_i) = 0$ for $i = 1,\ldots,n$, and

(A23) $F_i'(-A_i-S_i) - t = 0$ for $i = 1,\ldots,n$.

The analogous first order conditions for the general trading problem are

(A24) $F_i'(-A_i-S_i) + \lambda = 0$, for $i = 1,\ldots,n$, and

(A25) $S - \sum n_i S_i = 0$.

Suppose $A_1^*,\ldots,A_n^*$, $S_1^*,\ldots,S_n^*$, and λ^* solve the trading problem. Then let $-\lambda^* = t^*$, and the first order conditions to the tax problem are satisfied. Alternatively, suppose $A_1^*,\ldots,A_n^*$, $S_1^*,\ldots,S_n^*$, and t^* solve the tax problem. Then let $t^* = -\lambda^*$, and the first order conditions for the trading problem are satisfied (assuming t is positive).