Government Markets and the Theory of the *N*th Best

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Executive Summary

There is increasing interest in having the government create markets in property rights to allocate various resources, such as landing rights and environmental quality. The purpose of this paper is to provide some general guidelines for the more efficient organization of such markets. The analysis examines the problem of devising efficient government markets when there are multiple inputs to the production process subject to different taxes and regulations. It develops some simple rules of thumb for the design of markets and examines how they can be applied to problems involving regulated utilities.

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Robert W. Hahn

1. Introduction

Government markets are markets created by some level of government to allocate a particular commodity. Examples include markets in taxi cab medallions, liquor licenses, emissions permits, and development rights. The purpose of this paper is to provide some general guidelines for the more efficient organization of such markets. Efficiency is defined narrowly to correspond to cases in which a given target is achieved in a less expensive manner or aggregate profits of firms increase -hence the use of the term "N th" best.

Interest in the application of government markets to environmental problems has increased dramatically in the last few years (Stavins, 1988; U.S. EPA, 1991). For example, the 1990 clean air legislation calls for markets aimed at reducing sulfur oxide emissions by 10 million tons annually. These reductions are expected to come primarily from electric utilities. In addition, there has been growing interest in limiting utility carbon dioxide emissions through the use of a market (Dudek and LeBlanc, 1990).
Most economists believe a market for improving environmental quality will be preferable to more traditional command approaches because plant managers are in a better position to make pollution control decisions than government bureaucrats or politicians. For example, a simulation of a market to reduce sulfur oxide emissions that contribute to acid rain suggests that a national market could potentially save billions of dollars compared with traditional command-and-control approaches (Hahn, 1989). While such savings are possible, actual savings will depend on the structure of the market and the economic environment in which plant managers operate. Consider, for example, the case of utilities. Public utility regulation varies dramatically across states. Moreover, regulated utilities do not necessarily face economic "signals" that will lead to efficient production of output (Averch and Johnson, 1962; Joskow, 1974). Such signals typically come in the form of distorted prices, output requirements and revenue or profit constraints. More recently, it has been recognized that such signals may make it difficult to design regulatory schemes in which environmental objectives will be met in a cost effective manner (Hahn and Noll, 1983).

The primary contribution of this paper is to provide some relatively simple rules of thumb for addressing the question of efficient institutional design. These rules are derived from a model of firm behavior that distinguishes firms on the basis of the effective taxes or subsidies faced for inputs and outputs. The rules illustrate the kinds of price signals that are needed to induce utilities and other businesses to make more efficient decisions when subject to government markets. For illustrative purposes, the analysis is developed in the context of environmental markets.

The formal analysis is applicable to a broad range of problems in which the government limits the use of a particular commodity and allows trading in the rights to that commodity. This analysis is the first to consider the problem of

devising efficient government markets when there are multiple inputs to the production process subject to different taxes and regulations. It is also the first to develop formal results on improving efficiency in government markets (as opposed to simply identifying conditions for achieving an efficient solution).

The next section of the paper examines how contributions in the literature can help define an approach to the problem that is both tractable and useful. Section 3 develops a formal model, which yields some general guidelines for market design. Section 4 elaborates on the policy implications of the model and explores the implications of relaxing some of the model's assumptions. Conclusions and areas for future research are presented in Section 5.

2. Using the Literature to Construct a Modeling Approach

The model developed in the subsequent section will treat the firm's problem as one of selecting an optimal level of inputs and outputs subject to a set of prices and taxes that are exogenously determined. The model builds on two separate literatures. Insofar as the problem is characterized as one of "optimal taxation," the analysis is a direct outgrowth of the public finance literature (see, e.g., Atkinson and Stiglitz, 1980). This literature generally focuses on the development of a set of efficient taxes subject to a constraint that a certain level of revenues be raised by the government (Baumol and Bradford, 1970). This paper simplifies the second best problem by focusing on taxation rules that generate efficiency gains in a single market with no explicit revenue constraint. The rationale for relaxing the revenue constraint is that many government markets, most notably environmental markets, are not typically organized with the primary intent of raising a specific level of revenues.

A second relevant literature relates to the design of government markets. My concern here is with a subset of this literature that directly addresses problems related to distortions that can arise through utility regulation, and how such regulation might affect the behavior of firms in government markets. Subtle problems arise in defining the nature of public utility regulation. This regulation is important for environmental markets because utilities are often key players in such markets. There is a large literature on the economics of public utility regulation that suggests why utilities will choose an inefficient mix of inputs (Averch and Johnson, 1969). Averch and Johnson note the possibility of a capital bias. Subsequent research explores biases associated with other inputs, such as fuel (Isaac, 1982). This literature is ably reviewed by Joskow and Noll (1981), who note the limitations of the Averch- Johnson model and suggest why the results concerning excessive use of capital inputs may not have much practical relevance. The problem with deriving categorical results about the behavior of electric utilities is that they operate in a complex regulatory environment. For example, Joskow and Schmalensee (1986) argue that regulatory lag and prudency reviews are critical aspects of the utility rate making process. Regulatory lag could give utilities an incentive to cost minimize. Prudency reviews could make utilities think twice about gold-plating investments or making unnecessary expenditures on labor and energy inputs. There has been little empirical evidence to support the existence of either capital biases or fuel biases (Joskow and Rose, 1989).

Several authors have examined the relationship between environmental regulation and industry structure (Lee, 1975 and Oates and Strassman, 1984). This literature has tended to focus on the efficiency impacts of effluent fees. For example, Oates and Strassmann argue that the potential gains from a system of judiciously designed effluent fees are large, even in the presence of monopoly. They also identify conditions under which firms that do not maximize profits produce pollution control in an efficient manner.

The relationship between utility regulation and environmental regulation has been explored by Hahn and Noll (1983) in the context of a market for reducing sulfur oxides emissions. They argue that it is important for utilities to have an adequate incentive to sell permits, and that some approaches to ratemaking may not provide sufficient incentive because, in some situations, the entire proceeds of the sales would go to ratepayers with a commensurate reduction in the rate base. As a way of circumventing this problem, the authors argue for the use of a zero revenue auction that would establish the price of a permit for rate making purposes.

More recently, two papers have used formal models to capture salient aspects of utility behavior in a tradable permits market (Bohi and Burtraw, 1991; Hobbs, 1990). Bohi and Burtraw examine the trade-off utilities face between investing in capital equipment, such as scrubbers, and purchasing emission permits. The authors argue that these investments should be treated symmetrically in order to achieve efficiency. Moreover, when utilities do not have an incentive to limit the use of some subset of inputs, the authors find that regulation that encourages a cost- effective outcome in the environmental market becomes very difficult to implement. Hobbs examines the incentive that utilities may have to "hoard" permits (i.e., retain some positive amount for the future). Hobbs argues that hoarding can be a rational response to uncertainty and illustrates how different decision rules and degrees of risk aversion on the part of utilities will affect the level of hoarding.

This brief literature review suggests that we understand relatively little about the actual behavior of utilities and even less about how they are likely to behave in a an environmental market created by the government. On the basis of what is known, it is not unreasonable to presume that utilities minimize costs more or less like other firms, but often face different prices -- for example, the price of output may be set by the public utility commission and utilities may then be required to meet customer demand at that price.

Many of the constraints faced by utilities in the real world can be usefully modeled as effective taxes or subsidies. For example, an allowed rate of return that exceeds the market rate of return can be thought of as a subsidy, as can a rule that allows firms to pass through all fuel costs to ratepayers. Taxes on income to shareholders also would fall naturally within this framework.

The analysis proceeds on the assumption that it is instructive to model policy instruments affecting regulated and non-regulated firms as effective taxes or subsidies. These taxes and subsidies can be interpreted as a kind of reduced form. The strength of such a

modeling approach lies in its simplicity and generality; the weakness is that it glosses over some of the finer structure involved in regulated firm behavior. Given our rather limited understanding of this formal structure, however, taking such a straightforward approach seems sensible.

3. Modeling a Government Market in an N th Best World

This section develops a one period model of competitive firms participating in a permit market for emissions. All firms are assumed to act as price takers in all markets in which they participate. For simplicity, I assume that firms are engaged in two activities: producing output and emissions. Firms are distinguished by the taxes they incur on output, inputs and the permit market. Taxes should be interpreted broadly. They could correspond, for example, to a shadow price associated with a particular regulation. Taxes are assumed to affect the equilibrium price in the permit market, but taxes are not assumed to affect any other output or input prices.

Each firm maximizes profits within some tax structure.[1] A representative firm faces the following problem:[2]

Maximize $(1 - t)\, p\, f(x,E) - \Sigma\, (1 - t_i)\, w_i x_i - (1 - t_e)\, e\, E$, where[3]

$X_1, ..., X_n, E$

t = tax on revenue;

t_i = tax credit for i th input, i = 1, ..., n;

t_e = tax credit for an emission permit;

p = price of output;

$x = (X_1, ..., X_n)$ -- inputs to production, x_i = amount of i th input;

E = emission level, also treated as an input and assumed to equal the number of emission permits held by the firm;

$w = (W_1, ..., W_n)$ -- the input price vector, w_i = price of i th input;

e = price of an emission permit; and

f(x, E) = production function.

The first-order conditions for the representative firm are:

$(1-t)\, p\, f_i - (1 - t_i)\, w_i = 0$ for all i = 1, ..., n, and

(1)

$(1 - t)\, p\, f_E - (1 - t_e)\, e = 0$,

where f_i and f_E denote partial derivatives of f with respect to x_i and E, respectively. The first order conditions say that the after tax marginal product should equal the after tax input costs.

There are m firms, each of which can have different production and emission functions. In the interest of notational simplicity, I introduce superscripts related to particular firms only when necessary. The production function is strictly concave and twice differentiable, and the emissions function is strictly convex and twice differentiable.

These functions can differ across firms. Marginal products are assumed to be positive. The profit function is assumed to be strictly concave and twice differentiable, which is consistent with the previous assumptions concerning the production function and the emissions function.

The emissions permit market is competitive and has a total of A permits. In equilibrium, the permit price, e, is set so that the sum of emissions from all firms just equals A. The permit price is assumed to be positive (i.e. the emissions permit market is binding). It is given exogenously, but will vary with the particular tax system so as to equate the supply and demand for permits.

The N th best prices are defined as p and w, which are taken to be exogenous and fixed. That is, they are assumed not to vary with changes in the tax structure. They may be set by a regulator or result from the interaction of supply and demand.
There are two basic problems. One is to identify an efficient tax structure at the N th best prices and the second is to identify changes in the tax structure that will lead to a more efficient outcome at such prices.[4] The following theorem provides one characterization of an efficient tax structures.[5]

Theorem 1: Suppose t € [0,1) and t_e € [0,1). Let $t = t_1 = t_2 = ... = t_n$. Then, assuming there is an interior maximum, and the profit function has a unique maximum, the firm's input choices are independent of the particular choice of t. Moreover, emissions and output are produced efficiently.

This theorem says that if taxes are equal on all other inputs and output, then the tax on emissions can be set independently. This is a direct consequence of the assumption the supply of permits is perfectly inelastic. It is a natural extension of the idea that a tax structure where all taxes are equal does not distort input choices. For example, if a $ .05 per pound tax on a permit to produce one pound of chlorofluorocarbons were introduced, and the old equilibrium price were $ 1.00, then one would expect the new equilibrium price to be around $ .95 if the pre-tax demand for permits did not change. Thus, the reduction in price just equals the size of the tax and firms do not change any of their decisions. Congress and the Administration seem to have caught on to this basic idea. In 1989, they implemented taxes on chlorofluorocarbons, largely as a device for raising revenues.[6]

Note that the theorem does not directly address the issue of revenues. Again, to use the preceding example, suppose the tax were $.50 instead of $.05. The revenue collected by the government would increase without affecting allocative efficiency. This example shows how the tax structure in this theorem is compatible with a wide range of revenues. More generally, because input and output decisions depend here only on relative prices, changes in absolute levels of taxes can affect revenues without affecting allocative efficiency, so long as relative prices remain unaffected. This characteristic of the model also applies to the findings developed below.

In reality, the tax structure across inputs and outputs will rarely be equal for all firms. Thus, it is instructive to consider what sorts of changes might lead to greater efficiencies with different tax structures. The subsequent analysis will consider marginal and non-marginal changes to the tax structure, which can be achieved through a continuous transformation of the relative taxes.

Consider a case where the firm selects a level of output prior to choosing an emissions control strategy. Such an example would be a reasonable approximation when emission control costs represent a small fraction of the total cost of doing business. It would also be relevant to many short-term decisions faced by utilities, where they must minimize the costs of supplying a given level of output. In this case, it is possible to develop a useful theorem about the relationship between the tax structure and efficiency.

Theorem 2: Suppose choices concerning output and emissions are separable in the sense that output levels are chosen prior to making decisions on emissions. Let $(X_1,\ldots, X_n, E)$ define the set of inputs. Suppose initial tax credits are given by the vector $(t_1,\ldots,t_n, t_e)$ and initial tax ratios are defined by the vector $(r_1, ..., r_{n-1})$ where $r_i = (1 - t_i) / (1 - t_n)$. Let new tax ratios be defined by the relationship:
$r_i' = r_i + s (1 - r_i)$ for i = 1, ..., n-1, and t_e' € [0,1).

Assume s € [0,1]. Moreover, assume that the solution to the original tax problem is different from the N th best problem. Define real production costs as w*x. Then as s increases, real production costs decrease.

Note that since output is fixed, the decline in real production costs can be attributed to an equal decline in pollution control costs. The idea behind the theorem is illustrated in Figure 1 for a single firm. Suppose the firm minimizes the costs of achieving a given level of emissions, defined by the isoquant $E(X_1, X_2) = A$.[7]

The equilibrium associated with the initial tax is C, and is represented by the tangency of the budget line, denoted by T, with the isoquant. The equilibrium associated with the N th best prices is D and is associated with the budget line B. The theorem says that as the slope of the budget line moves from the slope associated with T to the slope associated with B, the cost of achieving the emissions target, A, is reduced. Note as the slope of the budget line changes in the stated manner, the equilibrium moves from C to D. The costs at C are characterized by the budget line B', which is parallel to B, but represents a higher level of expenditure measured at N th best prices. As C moves to D along the isoquant, costs decrease because the budget line measured at N th best prices shifts in towards the origin.
The theorem points out the importance of relative prices in the input markets. Thus for example, if there are two inputs for reducing emissions, the efficient solution is achieved

when the tax rates are the same, so that prices are proportional to N th best prices. The theorem also suggests that this result is independent of the permit tax so long as it does not exceed unity. Finally, the theorem suggests that efficiency will be enhanced as the relative costs of inputs converge to actual opportunity costs, even if the N th best result is not achieved. Thus, unlike most second-best results, the theorem provides important guidance on moving towards more efficient policies even when the efficient policy is not achieved. The theorem does not, however, provide an explicit statement of how the tax structure should be changed to enhance efficiency. Indeed, there are a wide array of tax structures that are compatible with increases in efficiency. These are implicitly defined by r_i'. For the special case in which all input taxes converge to one of the initial taxes, the following result obtains:

Corollary 1: Suppose that input taxes $t_1, ..., t_{n-1}$ converge to t_n in accord with the following formula:

$t_i' = t_i + s(t_n - t_n)$ for i = 1, ..., n-1,

t_n € [0,1), s € [0,1] and t_e' € [0,1). Then as s increases, real production costs decrease.

The proof of the corollary follows from showing that this explicit mapping for taxes is consistent with Theorem 2. Note that the selection of the input tax, t_n was arbitrary, and the result also applies to $t_1, ..., t_{n-1}$. Thus, convergence to any one of these taxes would result in lower real production costs and pollution control costs.

The preceding results assume that each firm faces the same tax structure. In reality, firms participating in government markets will frequently face different tax structures. For example, in the case of an emissions permit market, private firms and public utilities might face different taxes associated with permits. Different taxes for different firms will in general mean that the marginal control costs are not equated. The idea is illustrated in Figure 2, which shows marginal costs for two firms, 1 and 2. Abatement for firm I is measured from the left, while abatement for firm 2 is measured from the right. The marginal costs are denoted by MC_1 and MC_2. Note that, by construction, total abatement remains constant in the diagram at some specified level, assumed to correspond to emissions of A. The initial equilibrium is presumed to occur at A^0. From the standpoint of reducing overall costs, it would be preferable to move to another point, such as A^1, where overall control costs are reduced. A* represents the point where overall costs are minimized, since marginal costs are equated across sources.

The problem is to devise a taxation scheme that enhances efficiency when the initial tax structure for firms is different. The following lemma sheds some light on the problem.

Lemma 1: Suppose firm j's tax structure is given by $(t, t_1, ..., t_n, t_e^j)$, so that each firm faces the same tax in a given market, with the exception of the market for permits. Assume that not all firms face the same initial tax for permits. Let

$$t_e^{jN} = t_e^j + s\,(K - t_e^j),\ s \in (0,1] \text{ and } K \in [0,1)$$

represent new taxes in the permit market. Then, under the new tax structure, it can be shown that aggregate firm profits, measured at prices $((1-t)\,p,\ (1-t_1)\,w_1,\ \ldots,\ (1-t_n)\,w_n)$, increase.

The proof is based on two ideas. First the derived demand for permits is downward sloping. Second, as taxes converge to some positive constant between 0 and 1, the marginal costs of abatement across firms tend to approach each other, and this results in increased profits.

Note that this lemma says that as emissions taxes converge to some positive constant, aggregate firm profits will increase. If output levels are selected prior to pollution control decisions, then it can be shown that direct expenditures on abatement will decrease.[8]
One problem with the lemma is that it does not apply to the case of N th best prices, but rather to the original set of after tax prices associated with the initial tax structure. A second problem with the result is that it assumes that all other taxes remain constant across firms. The following theorem addresses both these concerns.

<u>Theorem 3:</u> Suppose firm j's tax structure is given by $r^j * (t, t1, \ldots, t_n)$ and $t_e j$, where r^j is a scalar and $t = t_1 = \ldots = t_n$. Let $t_e^j{}' = 1 - [(1 - t_e^j) / (1 - r^j t)]$. If $t_e^j{}'$ are equal across all firms, then the solution is achieved. If $t_e^j{}'$ are not equal across all firms, then the transformation

$$t_e^{jN} = t_e^j{}' + s\,(K - t_e^j{}'),\ s \in (0, 1] \text{ and } K \in [0, 1)$$

results in an increase in total firm profits measured at N th best prices.

As s approaches 1, t_e^{jN} approaches K. This implies that the ratio of the fraction of permit cost paid by the firm to the fraction of other input costs paid by the firm converges to a constant, where this ratio is initially defined by $(1 - t_e^j) / (1 - r^j t)$. In other words, the budget line defining the trade-off between permits and all other inputs converges to the same slope across firms. Note that efficiency requires that $t_e^j{}'$ are equal across all firms, but this means that firms can face different taxes on permits and other taxable items, so long as the preceding ratio is constant across firms. This result stands in sharp contrast to the result by Bohi and Burtraw (1991), who argue that cost recovery rules should be symmetric for permits and control technology, which in this model is equivalent to assuming that all taxes must be equal to achieve efficiency in the permit market. The reason for the difference in the results is that Bohi and Burtraw highlight the case of a single state regulator who cannot influence the market price of permits. Their result is

directly analogous to the small country theorem in international trade, which suggests that no tariffs are required for optimality.

The proof follows from Lemma 1 through a suitable transformation of the taxes. If output is selected prior to making decisions about pollution control, then the theorem implies that aggregate expenditures on pollution control decrease with changes in the appropriate taxes. The key idea contained in the theorem is that it is relative taxes that matter. As the ratio of the fraction of permit cost paid by the firm to the fraction of other input costs paid by the firm converges for all firms, then the equilibrium moves from, say, A^0 in Figure 2 to A^1 and eventually to the optimum, A *.

4. Policy Implications

The model in the previous section was used to develop three results. First, the tax structure on permits does not matter when other taxes are set equal to each other. Second, if output is fixed, pollution control costs will decline as the relative cost of inputs faced by the firm are brought into line with their real opportunity costs. Finally, if the tax structure is identical for inputs and outputs within a firm, but firms face different relative costs for inputs and permits, profits will increase as these relative costs converge across firms.

Within the context of the model, there are several policy implications that flow from the analysis. I begin by discussing these implications and then explore the implications of relaxing some assumptions of the model. Finally, I offer some observations on the design of government markets.

The basic message of the model is that designing an optimal tax structure for permits is not a particularly challenging exercise if the other relative prices are "right." The model provides no justification for distinguishing permits based on their intended use. For example, permits that substitute for cleaner fuel should not be treated any differently than permits that substitute for hardware, such as scrubbers. Indeed, theorem 2 highlights the value of trying to equate the tax on capital and variable inputs in promoting efficiency in the permit market.

A common problem in utility regulation is that there are multiple jurisdictions, typically defined by state regulatory commissions. Theorem 3 suggests that it is appropriate to adjust relative taxes on emissions and other inputs so that the ratio of the fraction of permit cost paid by the firm to the fraction of other input costs paid by the firm converges across jurisdictions. This, of course, is easier said than done. The taxes used in this model are not readily identifiable parameters in the context of the utility rate making process. Nonetheless, I believe they provide a useful way of thinking about the problem. Moreover, public utilities, like other private companies, can be expected to respond to different effective tax rates or shadow prices.

Theorem 3 also has relevance to devising tax structures across public utilities and private companies. The moral is the same as with the problem of multiple state regulation --

ensure that the budget line defining the trade off between permits and all other inputs converges to the same slope across firms.

The model as constructed assumes that all firms produce the same output. This was done for the sake of convenience. The basic results can be extended to the case where firms produce multiple outputs. The critical features driving the results are the shape of the output and emission functions and the assumption that the market price for permits adjusts in response to the tax structure.

One potentially important case not considered in the preceding analysis is when utilities have an incentive to make excessive use of inputs. For example, if utilities can pass on 100 percent of fuel costs or their effective cost of capital is less than the actual rate of return, serious inefficiencies could result. The nature of the bias (e.g., towards scrubbing or cleaner fuels) would depend on the relative returns from these activities. In the context of the model, this situation would correspond to the case where an input was effectively subsidized so that a firm would have an incentive to maximize its use.

In reality, as Joskow and Schmalensee (1986) note, actual regulatory approaches include regulatory lags and prudency reviews. Lags would tend to reduce the effective subsidy; prudency reviews could also serve this function. Nevertheless, if there were an effective subsidy, the regulator could pursue two basic strategies. The first would be to remove the effective subsidy, so the utility incurs some costs associated with using a particular input. The second would be to monitor the usage of the input carefully, and in the extreme case, dictate the level of the input's use. The extreme case should be considered only after careful consideration has been given to options that create a proper regulatory environment in which the firm can make such decisions, since the firm typically has better information than the regulatory about its own costs.

The evidence for subsidies is relatively weak. Nonetheless, if effective subsidies were thought to be a serious problem, then another way of addressing this issue is to remove barriers to competition in the electric generation sector through further deregulation. This policy may not be justified solely to rationalize a government market in emissions, but makes economic sense in its own right (Stelzer, 1982; Joskow and Schmalensee, 1983). Without a significant restructuring of the utility industry, the presence of significant effective subsidies could make it very difficult to have a reasonably efficient market in emission permits.

The preceding analysis presumes that regulators can exert some control over the regulatory treatment of expenditures on pollution control. If regulators are severely constrained in how they can adjust effective taxes of inputs, then the basic structure of the problem needs to be revised. Even if the regulator were highly constrained, however, it is difficult to imagine actual situations in which the asymmetric treatment of permits based on sales and purchases could be justified on efficiency grounds.

One important issue for regulators is the treatment of gains and losses from the sale and purchase of permits. In the static model used here, there is only one price for permits, and

all permits are used. This feature of the model makes it relatively easy to tax permits at some specified rate, and as noted in Theorem 1, the precise rate is of little consequence. The situation becomes less clear in a multiperiod model and deserves further study. In this latter case, it is not obvious how to specify an appropriate welfare criterion, since it is unreasonable to assume that prices will be stable over time. Indeed, a crucial part of the problem is that firms will experience gains and losses based on their strategies in the allowance market and expenditures on pollution control. Common sense suggests that utilities should receive some of the gains as well as the losses associated with their investment strategy related to pollution. Whether this sharing should be symmetric will depend on the particular regulatory environment.[9]

The model tacitly assumes that transactions costs are negligible. This is a critical issue in the design of government markets (see, e.g., Hahn, 1989), but is beyond the scope of this paper. Suffice it to note that the government can reduce transactions costs in several ways -- perhaps most importantly, by specifying well-defined property rights.
While the model does not include any institutions other than a highly stylized permit market, such institutions can be important in actual applications. Making more effective use of existing institutions can be a low-cost way of helping to achieve a more efficient allocation of emission permits. In the case of electric power, there are already well-defined markets for exchanging electricity over the short and long term. It would be a relatively modest step for such markets to include the incremental cost of pollution control, as it is reflected in the price of a permit, in the cost of selling power. In this way, the external cost of pollution would be internalized.

Some of the opportunities and challenges in implementing the basic insights of this model can be seen by examining a particular problem. Take, for example, the case of designing a market in sulfur oxides emission permits aimed at reducing acid rain. The Federal Energy Regulatory Commission and the U.S. Environmental Protection Agency have some control over the rules governing this market at the federal level; however, ultimately, state public utility commissions must determine how these emission permits can be treated by utilities that participate in the market.

A critical problem in market design is to determine what type of guidance federal regulators and other interested parties might want to offer state regulators. One approach might require that federal agencies ask the states to identify the effective taxes associated with their regulations, and then apply some variant of the theorems to enhance efficiency in the permit market. This approach has the serious drawback that the effective taxes are difficult to identify, and thus it would be difficult to operationalize the model. An alternative approach would be to regulate pollution control decisions as if they were largely separable from output decisions. In the case of sulfur dioxide emissions, this could be done relatively easily for control equipment, such as scrubbers, as well as for fuels with varying sulfur contents. The problem would be less tractable for new generating plants. Assuming some measure of variable and fixed control costs could be defined, it would be relatively straightforward to develop rules that would enhance efficiency in the permit market. Shareholders could be asked to pay for some fraction of direct pollution control costs and some fraction (not necessarily the same) of permit costs.

Ratepayers would be responsible for the remainder of the costs. If the minimization of the sum of control costs and permit holdings were expected to result in "unfair" losses to shareholders, then some lump-sum transfer mechanism could be introduced. This second approach, while resting on the strained assumption concerning separability, has the distinct advantage that it is more easily implemented.

5. Conclusions and Areas for Future Research

This paper has developed some simple rules of thumb that should help guide policy makers and regulators in the design of government markets. If there is a single message of the paper, it is that relative prices matter, and that the tax structure should be used to adjust relative prices to achieve more efficient solutions. The relatively simple model structure permitted the development of some reasonably powerful rules of thumb. Richer models will undoubtedly yield results with more complex rules.

There are several potentially useful extensions of the model. For example, as noted above, the inclusion of a time dimension could help resolve issues related to allowance treatment when there are gains and losses. A second helpful extension would be to consider the case of multiple government markets. For example there could be several markets in emissions or a market in emissions and a market in output, such as electricity, where the demand is fixed over a short period of time. A cursory analysis of the multiple markets case suggests that a variant of the second theorem is applicable, albeit in somewhat weaker form.

The preceding analysis takes all prices as given, except the emission permit price. Clearly, in markets, such as the one for acid rain, fuel prices can be expected to adjust in response to the introduction of the market. Moreover, in markets that are not competitive, it may be unreasonable to assume that prices are given. This raises the difficult issue of defining an appropriate set of N th best prices. Addressing this problem may require a more complete specification of a welfare function involving standard consumer and producer surplus measures. Moreover, the model of N th best prices does not take into account potential deadweight losses associated with an increased tax burden. This analysis could be usefully extended by building on insights from the public finance literature. It can be expected to yield results similar to those contained in the literature on the second best.

While the analysis in this paper does not explicitly consider transactions costs, such costs are likely to be critical in assessing the role of government markets. It would be useful to develop a theory of transactions costs and government markets that parallels the theory developed here. A critical challenge is how to develop a formal analysis of transactions costs that has practical relevance.

This paper has focused on normative considerations. Having identified some efficiency-enhancing improvements, it would be instructive to explore the conditions under which they are likely to be adopted. This set of issues merits further consideration. A first step in pursuing such an analysis would be to carefully specify likely winners and losers from different tax policies (see, e.g., Meltzer and Richard, 1981).

Government markets are likely to be used more frequently for implementing public policies, particularly in the environmental arena. It is essential that we develop a better understanding of how to implement such markets in theory and practice. This paper will hopefully serve as an initial step toward framing the issues that need to be resolved.

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Appendix

The appendix provides proofs of the theoretical results.

<u>Theorem 1:</u> Suppose $t \in [0,1)$ and $t_e \in [0,1)$. Let $t = t_1 = t_2 = \ldots = t_n$. Then, assuming there is an interior maximum and the profit function has a unique maximum, the firm's input choices are <u>independent</u> of the particular choice of t. Moreover, emissions and output are produced efficiently.

<u>Proof:</u> The proof proceeds in two parts. First, the conditions for efficient joint production are defined. Then, these are shown to be consistent with individual profit maximization and independent of the choice of t.

The joint profit maximization problem is given by:

$$\underset{X^1,\ldots,X^m,\,E^1,\ldots,E^m}{\text{Maximize}} \quad \sum_j [p\, f_j(X_j, E_j) - w^*X_j]$$

$$\text{subject to:} \quad \sum_j E_j = A,$$

where w^*X_j is a dot product.[10]

This yields the Lagrangian:

$$L = \sum_j [p\, f_j(X_j, E_j) - w^*X_j] + \lambda\left(\sum_j E_j - A\right),$$

with the following first order conditions for an interior maximum.

$$pf^j_i - w_i = 0,\ i = 1,\ldots,n;\ j = 1,\ldots, m, \text{ and (Al)}$$

$$pf^j_E + \lambda = 0,\ j = 1,\ldots, m,$$

along with the constraint that aggregate emissions equal the number of emissions permits.

The first order conditions for the individual profit maximization problem are given by (1). Setting $t = t_1 = t_2 = \ldots = t_n$ implies that the first set of first order conditions defined by (Al) are satisfied, independent of the choice of t. To show that the second set of first order conditions in (1) is satisfied, set $e = -\lambda (1 - t) / (1 - t_e)$. Note again that these first order conditions are satisfied independent of the choice of t. This value of e guarantees the aggregate emission constraint is met, thus completing the proof.■

If decisions on emissions do not affect the level of output, the same proof applies to the case of emissions being produced efficiently (<u>i.e.</u>, at least cost.)

<u>Theorem 2:</u> Suppose choices concerning output and emissions are separable in the sense that output levels are chosen prior to making decisions on emissions. Let $(x_1,\ldots, x_n, E)$ define the set of inputs. Suppose initial tax credits are given by the vector $(t_1,\ldots, t_n, t_e)$ and initial tax ratios are defined by the vector $(r_1, \ldots, r_{n-1})$ where $r_i = (1 - t_i) / (1 - t_n)$. Let new tax ratios be defined by the relationship:

$r'_i = r_i + s(1 - r_i)$ for $i = 1, ..., n-1$, and t'_e € [0,1).

Assume s € [0,1]. Moreover, assume that the solution to the original tax problem is different from the N th best problem. Define real production costs as w*x. Then as s increases, real production costs decrease.

Proof:[11] It will be convenient for purposes of the proof to assume that there are n+1 inputs. The j th firm faces the following minimization problem under the new tax structure:

$$\text{Minimize} \sum_{X_{j,Ej}} (1 - t_{i'})\, w_i x_i^j \quad (1-t)\, e_{Ej} \qquad \text{(A2)}$$

Subject to: $f_{j(Xj, Ej)} = q^j$

where the summation is taken over i = 1, ..., n+1, q^j (j = 1, ..., m) is the level of output for firm j.

Define G(x) = Minimum

$$\sum_{X^1, ..., X^m, E^1, ..., E^m} j\, Ej$$

subject to: $\sum_j X_j = x$ and $f_j(X_j, E_j) = q^j$, j=1, ..., m.

Define Ei(Xi) implicitly as the minimum value of E_j that solves $f_j(X_j, E_j) = q^j$. Then, G(x) can be redefined as

$$G(x) = \text{Minimum} \sum_{X^1, ..., X^m} j\, Ej\, (X_j) \qquad \text{(A3)}$$

subject to: $\sum_j X_j = x$.

It can be shown that G(x) is strictly convex in x. First, note that E(X) is strictly convex. (The firm-specific superscripts have been dropped for simplicity.) From the definition of convexity, it suffices to show that

$E(\lambda X_1 + (1 - \lambda)X_2) < \lambda E(X_1) + (1 - \lambda) E(X_2)$ for all λ € (0,1) and $X_1 \neq X_2$.

Let $E(X_1) = E_1$, $E(X_2) = E_2$ and $E^* = E(\lambda X_1 + (1 - \lambda) X_2)$, which means that the claim is that $E^* < \lambda E_1 + (1 - \lambda) E_2$. Suppose not. Then

$$\lambda f(X_1, E_1) + (1 - \lambda) f(X_2, E_2) < f(\lambda (X_1, E_1) + (1 - \lambda)(X_2, E_2)).$$

$\leq f(\lambda X_1 + (1 - \lambda) X_2, E^*)$,

where the first line follows from the strict concavity of f and the second line follows from the assumption that the marginal product of E is positive. The preceding result violates the assumption that the output associated with E* is the same as the output associated with E_1 and E_2, which implies $E^* < \lambda E_1 + (1 - \lambda) E_2$ and, thus, E(X) is strictly convex.

To show G(x) is strictly convex in x, let x* and x** be distinct points in R^n and let X^1*, ..., X^m* be the solution to the minimization problem (A3) when x = x* and X^1**, ..., X^m** be the solution when x = x**. Then for all μ € (0,1)

$$G(\mu x^* + (1 - \mu) x^{**}) \leq \sum_j (\mu Xj^* + (1 - \mu)Xj^{**})$$

$$< \mu \sum_j Ej(Xj^*) + (1 - \mu) \sum_j Ej(Xj^{**})$$

$$= \mu G(x^*) + (1 - \mu) G(x^{**}),$$

where the first inequality follows from the definition of G, the second inequality follows from strict convexity and the final equality follows from the definition of G.

The solution to the preceding problem is the same as the following problem:

$$\text{Minimize} \sum_{X_1, ..., X_{n+1}} (1 - t'_i) w_i x_i \quad (A4)$$

subject to: G(x) = A,

where the summation is taken over i = 1, ..., n+1, and it is assumed that the solution to (A2) yields a total of A emissions.

The equivalency follows from a proof by contradiction. That is, assuming a solution to the first is dominated by a solution to the second leads to the conclusion that the solution to the first was not optimal and vice versa.

For notational convenience, let $X_{n+1} = y$, $X_{n+1} = t_y$ and $w_{n+1} = w_y$ and let (x*,y*) be the unique optimum. Note that G is a C² function on R_{n+1}, $(x,y) = (x_1,..., x_n, y)$ and $G_y \neq 0$ on the constraint set defined by G(x,y) = A. For (x,y) in the constraint set, let

$m_i = -G_i / G_y$, $i = 1, \ldots, n$,

where $G_i = \partial G / \partial X_i$ and $G_y = \partial G / \partial y$. Let $m_i = (m_1, \ldots, m_n)$ and $m^* = (m_{1*}, \ldots, m_{n*})$, where $m_i^* = m_i(x^*,y^*)$.

At the point (x*,y*) in the constraint set, the tangent plane satisfies:

$y - y^* = \Sigma m_i^* (x_i - x_{i*})$.

Define the value function, V, such that

$V(x,y) = w_y (y - \Sigma m_i^* x_i)$.

The proof proceeds in two parts. First, I note that the point (x,y) on the constraint set is determined by the "slope" m, and the

Limit $(x,y) = (x^*,y^*)$.
$m \dashrightarrow m^*$
That is, the dependence of (x,y) on m is continuous (in fact, it is C^1).

In the second part of the proof, I argue that variations in the slope towards m* decrease the value function. Consider a variation of the slope m of the form

$$m(\lambda) = m^* + \lambda v,$$

where v is defined so that all elements of the slope in the relevant range for λ are negative. Along such a variation, which yields corresponding values for (x,y), V

achieves an absolute minimum at the point $\lambda = 0$. In fact, $dV/d\lambda$ and λ always have the same sign (positive or negative).

First, consider the relationship between changes in the slope and the optimal point on the constraint set. The fact that G is strictly convex means that no two points in the constraint set have the same (or parallel) tangent planes. Thus, the slope m determines the point (x,y).
To show that the dependence is continuous, let (x*,y*) represent an arbitrary point on the constraint set. Since $G_y \neq 0$ at (x*,y*), the implicit function theorem says that the constraint set can be locally parameterized by a C^2 function

$$y = h(x_i, \ldots, x_n)$$

Then $m_i = -G_i / G_y = \partial h / \partial x_i$ for i = 1, ..., n, where $\partial h / \partial x_i$ is a C^1 function of x, and $\partial m_i / \partial x_k = \partial^2 h / \partial x_i \partial x_k$, i,k = 1, ..., n.
Since G is strictly convex, it follows that the n x n Hessian matrix $[\partial^2 h / \partial x_i \partial x_k]$ is positive definite. This can be shown by first noting that $G(X_1, \ldots, x_n, h(X_1, \ldots, x_n)) = A$, which follows from the definition of h. Differentiation of the above equation with respect to x_i yields:

$$\partial G / \partial x_i + (\partial G / \partial y)(\partial h / \partial x_i) = 0.$$

Differentiating this _expression with respect to x_k yields:

$$\partial^2 G / \partial x_i \partial x_k + (\partial^2 G / \partial x_i \partial y)(\partial h / \partial x_{k)} + (\partial^2 G / \partial x_k \partial y)(\partial h / \partial x) +$$
$$(\partial^2 G / \partial y^2)(\partial h / \partial x_i)(\partial h / \partial x_k) + (\partial G / \partial y)(\partial^2 h / \partial x_i \partial x_k) = 0,$$

which is true for all i and k.
Multiplying the preceding _expression by a_i and a_k, summing over i,k = 1, ..., n, and rearranging terms gives:

$$\Sigma i\, \Sigma k\, a_i a_k [\partial^2 G / \partial x_i \partial x_k + (\partial^2 G / \partial x_i \partial y)(\partial h / aX_k) + (\partial^2 G / \partial x_k \partial y)(\partial h / \partial x_i)$$
$$+ (\partial^2 G / \partial y^2)(\partial h / \partial x_i)(\partial h / \partial x_k)] = -(\partial G / \partial y)\, \Sigma i\, \Sigma k\, a_i a_k (\partial^2 h / \partial x_i \partial x_k).$$

Define the vector $a = (a_1, ..., a_n, \Sigma_i a_i (\partial h / \partial x_i))$. Then the quadratic form a [Gik] a^T, where T denotes transpose, is equal to the left hand side of the preceding equation. If $a \neq 0$, positive definiteness implies that the left hand side of the equation is positive. Since $\partial G / \partial y < 0$, it follows that $\Sigma i \Sigma k\ a_i a_k (\partial^2 h / \partial x_i \partial x_k) > 0$ when $a \neq 0$, which means that the Hessian of h is positive definite.
12
Because the Hessian of h is positive definite, the matrix of partials $[\partial m_i / \partial x_i]$ is non-singular. By the inverse function theorem, the function $m = m(x_1,..., x_n)$ can be inverted to obtain a C^1 function $x = x(m_1, ..., m_n)$. Thus, $(x,y) = (x(m),h(x(m))$ depends continuously on m and it is differentiable with respect to m, thus showing the first step in the proof.

To show how the value function varies with X, define
$V(m(\lambda)) = w_y [h(x(m(\lambda))) - \Sigma\ m_i^* x_i (m(\lambda))]$.

Then

$dV/d\lambda, = w_y [\Sigma i\ \Sigma k\ ((\partial h /\partial x_i) - m_i^*) (\partial x_i / \partial m_k) (\partial m_k / d\lambda)$.

From the expression for $m(\lambda)$, it follows that $\partial m_k / d\lambda = V_k$. Also recall that $(\partial h / \partial x_i) =$ mi, which equals $m_i^* + \lambda v_i$ when evaluated at $m_i(\lambda)$. Thus,

$\partial h / \partial x_i - m_i^* = \lambda v_i$, and the _expression for $dV/d\lambda$, simplifies to

$dV/d\lambda = w_y [\lambda\ \Sigma i\ \Sigma k\ ViVk\ (\partial x_i / \partial m_k)]$.
By the inverse function theorem, the matrix $[\partial x_i / \partial m_k]$ is the inverse of the matrix $[\partial m_k / \partial x_i] = [\partial^2 h / \partial x_i \partial x_k]$, which is positive definite, so $[\partial x_i / \partial m_k]$ is positive definite. Hence, $\Sigma i\ \Sigma k\ ViVk\ (\partial x_i / \partial m_k) > 0$ for all $v \neq 0$.

Thus, if $\lambda > (<) 0$, $dV/d\lambda > (<) 0$ and when $\lambda = 0$, $dV/d\lambda = 0$, which completes the second part of the proof.
It remains to show that the mapping defined in Theorem 2 corresponds to an appropriate λ and v in the equation $m(\lambda) = m^* + \lambda v$. The tangent plane associated with $m(\lambda)$ is given by the equation $y - \Sigma\ m_i (\lambda)\ x_i = c$, where c is a constant determine by the actual point of tangency with G(x,y). After dividing the objective function in (A4) by w_y and $(1 - t_y')$ the objective function becomes
$y + [(1-t_i')/(1 - t_y')] (w_i /w_y) x_i = y + \Sigma - r_i' (w_i /w_y) x_i$.

It suffices to equate the coefficients of x_i in the tangent plane with the coefficients of this modified objective function, since the coefficient for y is 1. First note that $m_i^* = -w_i / w_y$. This follows from examining the first order conditions associated with (A4) and the definition of m_i^*. Equating the coefficients for x_i yields:

$r_i' (w_i/w_y) = (w_i/w_y) - \lambda V_i$.

Substituting for r_i' yields:

$[r_i + s (1 - r_i)] (w_i/w_y) = (w_i/w_y) - \lambda v_i$.

For given v, s is a linear function of λ. When s = 1, let λ = 0 and the equation will hold for any v_i. The choice of vi is somewhat arbitrary, but could be defined so that when s = 0, λ = 1. In this case

$v_i = (w_i/w_y) (1 - [r_i + s (1 - r_i)])$ i = 1, ..., n, which completes the proof. ■

<u>Corollary 1:</u> Suppose that input taxes $t_1, ..., t_{n-1}$ converge to t_n in accord with the following formula:

$t_{i'} = t_i s (t_n – t_i)$ for i = 1, ..., n-1,

t_n € [0,1), s € [0,1] and t_e' € [0,1). Then as s increases, real production costs decrease.

<u>Proof:</u> It suffices to show that this is a special case of the theorem. Let $t_n = t_y = k$. Then, from the theorem:

$r_i' = (1- t_i) / (1-k) + s (1-((1- t_i) / (1-k)))$

$= (1 - t_i + (1 - k) s - s (1 - t_i)) / (1-k)$

$= (1 - t_i - s (k - t_i)) / (1-k)$

$= (1 - (t_i + s (k - t_i)) / (1-k)$

$= (1 - t_i') / (1 - t_y')$.

The first equality follows from the definition of r_i' and t_i'. The second equality follows from making 1 - k the common denominator. The remaining equalities follow from simplifying of terms, and in the case of the last equality, substitution.[13] ■

<u>Lemma 1:</u> Suppose firm j's tax structure is given by $(t, t_1, ..., t_n, t_e j)$, so that each firm faces the same tax in a given market, with the exception of the market for permits. Assume that not all firms face the same initial tax for permits. Let

$t_e^{jN} = t_e^j + s (K - t_e^j)$, s € (0,11 and K € [0,1)

represent new taxes in the permit market. Then, under the new tax structure, it can be shown that aggregate firm profits, measured at prices $((1- t)p, (1-t_1)w_1, ..., (1 - t_n)w_n))$, increase.

Proof: The proof is based on two ideas. First the derived demand for permits is downward sloping. Second, as taxes converge to some positive constant between 0 and 1, the marginal costs of abatement across firms tend to approach each other, and this results in increased profits. To show that the demand for permits is downward sloping, consider the general profit function defined by:

$$\pi(p,w,e) = \underset{E}{\text{Maximize}}\ p\, f(x, E) - \Sigma\, w_i x_i - e\, E\ x_1,\ldots,x_n,$$

The claim is that $\partial E/\partial e < 0$. The proof follows directly from Varian (1978, pp. 30-32). First note that the profit function is strictly convex in p,w, and e given the strict convexity of the profit function in x. Let x*, E* be the profit maximizing level of inputs associated with (p*,w*,e*). Define the function
$g(p,w,e) = \pi\,(p,w,e) - (p\, f(x^*, E^*) - wx^* - e\, E^*$.
Note that the function is strictly convex, twice differentiable and achieves its unique minimum at (p*,w*,e*). The first order condition for a minimum implies:
$g_3\,(p^*,w^*,e^*) = \pi_3\,(p^*,w^*,e^*) - E^*$ = where the subscript denotes the partial derivative with respect to that argument. Since this argument holds for all (p*,w*,e*) this yields the relationship:

$$E(p,w,e) = \pi_3\,(p,w,e).$$

Differentiation with respect to e
yields $E_3\,(p,w,e) = \pi_{33}\,(p,w,e)$.
Since $\pi_{33} < 0$ (by strict convexity and differentiability), this yields $\partial E/\partial e < 0$, which is the desired result.

Essentially the same proof can be used to show that the derived demand for permits is downward sloping when output levels are selected prior to making pollution control decisions. In this case the problem becomes a cost minimization problem. The proof also extends to the case of multiple outputs and multiple pollutants, when there is one market associated with a particular pollution type and the remaining pollutants are regulated using quantity constraints.

Consider the problem of modeling the impact of changes in the tax structure. It will be useful to introduce some notation that allows the proof to proceed in abatement space. Define the old emissions equilibrium as $E^O = (E^{10},\ldots, E^{mo})$; and the new emissions equilibrium as $EN = (E^{1N},\ldots, E^{mN})$. Let baseline emissions equal $EB = (E^{1B}\ldots, E^{mB}$. Abatement is then defined as the vector $A = (A^1\ldots, A^m)$, and A EB - E, the

difference between baseline emissions and some specified level of emissions. Let the change in abatement between the old and new be defined by the vector $\Delta A = A^N - A^O$.

The inverse demand for permits can be characterized by the function $e(E^j *)$, where * = 0 or N. Marginal costs of abatement for firm j, or in this case, marginal profits foregone by abatement, are given by the formula:

$MC^j * = MC^j *(A^j *) = e(E^j *)$, where * = 0 or N.[14] The relationship between the marginal costs of abatement and permit demand is straightforward (see, e.g., Hahn, 1984).
The problem is to show that the increase in total costs (decrease in total profits) resulting from the change in abatement is less than the decrease in total costs (increase in total profits). Let the increase in total costs for firm j be denoted by ΔTC^j, assuming it actually experienced a cost increase (i.e., $\Delta Aj > 0$). Let ΔTS^j be the increase in total savings for firm j if it experienced a decrease in cost (i.e., $\Delta Aj < 0$). Then the claim is:

$\Sigma\ \Delta TCi < \Sigma\ \Delta TS)$.

$\Delta Aj > 0\ \Delta Aj < 0$

Assume that the original costs can be ranked in ascending order so that $MC^{10} < ... < MC^{mo}$. That is, $t_e^1 > ... > t_e^m$. It is then possible to show $MC^{10} < MC^{1N} <... < MC^{mN} < MC^{mo}$. In words, this says that the effect of the new tax adjustment is to raise the lowest marginal cost, reduce the highest marginal cost, and preserve the same ordering of marginal costs. Define the old market clearing price of permits as e^o and the new price as e^n. Then, $MC^{jo} = (I - t_e j)\ e^o$ and $MC^{jN} = (1 - t_e^{jN})\ e^n$ for all j = 1,…, m. If $MC^{io} < MC^{jo}$, this implies $t_e i > t_e j$; similarly if $MC^{iN} < MC^{jN}$, this implies $t_e^{iN} > t_e^{jN}$. Thus, it suffices to show $t_e i > t_e j$ implies $t_e^{iN} > t_e^{jN}$ to show that the ordering is preserved

$t_e i > t_e j$ iff

$(1 - s)\ t_e i > (1-s)\ t_e j$ iff

$(1 - S)\ t_e i + sK > (1-S)\ t_e j + sK$ iff

$t_e^{iN} > t_e^{jN}$,

where the second line follows from multiplication by a positive number, the third line follows by adding like terms to both sides and the fourth line follows by definition.

It remains to show that $MC^{10} < MC^{1N}$ and $MC^{mN} < MC^{mo}$. To demonstrate these inequalities, I first show that rate of increase from MC^{iN} to $MC^{j+1,N}$ is less than the rate of increase from MC^{jo} to $MC^{j+1,O}$ for j = 1, ..., m-1. C^{j+1},N = $[(1 - t_e j+1,^{N})/(1 - t_e j^{N})]$ MCj^{N}. It suffices to show that $(1 - tej+1,^{N}) / (I - t_e j^{N}) > (1 - t_e j+1,0)/(1 - t_e j^{O})$.

$$(1 - t_e j+1,^{N}) / (1 - t_e j^{N}) = [1 - (1- s) t_e^{j+1,O} - sK / [1 - (1- s) t_e^{jO} – sK]$$

$$= [(1 - s) - (1- s) t_e^{j+1,O} + s – sK] / [(1 - s) - (1- s) t_e^{jO} +s – sK]$$

$$= [1 - t_e^{j+1,O} +s (1 - K) (1-s)-^{1}] / [(1 - t_e^{jO} + s (1 - K) (1-s) -^{1}]$$
$$> (1 - t_e^{j+1,O})/(1 - t_e^{jO}).$$

The first equality follows from direct substitution; the second results from adding and subtracting s in both the numerator and denominator. The third equality results from dividing numerator and denominator by (1 - s). The final inequality follows from recognizing that the previous equality yields an _expression of the form (a + J) / (b + J), where 0 < (a / b) < 1 and J > 0 ($a \equiv 1 - t_e^{j+1,O}$, $b \equiv 1 - t_e^{jO}$ and $J \equiv s (1 - K) (1-s) -^{1}$.

Having shown the rate of increase is less, now suppose $MC^{10} > MC^{1N}$. This would imply that abatement levels are less than or equal to the initial abatement levels, with the strict inequality holding for all but firm 1. This would thus lead to higher costs (lower profits) than necessary, violating the assumption that profits are maximized. Thus, it must be the case that $MC^{10} < MC^{1N}$.

Now suppose $MC^{mN} \geq MC^{mo}$. This would lead to insufficient abatement, which would violate the aggregate emissions constraint. This establishes the relationship for new and old marginal costs. Now, note that

$$\sum_{\Delta Aj > 0} \Delta TC^{j} < \sum_{\Delta Aj > 0} MC^{jN} \Delta Aj \quad (A7)$$

$$< \sum_{\Delta Aj \leq 0} MC^{jN} |\Delta Aj|$$

$$< \sum_{0}^{j} \Delta TS \, \Delta Aj <_{}$$

The expression on the right hand side of the first inequality is an upper bound on total cost increases because the marginal abatement cost function is positively sloped (i.e., permit demand functions are downward sloping), and because the marginal cost at the new abatement level for those firms with cost increases represents an upper bound on costs per unit of new abatement for each firm. The second inequality follows from three facts. First, the abatement increase associated with marginal cost increases just equals the abatement decreases associated with marginal cost decreases. Second, the new marginal costs are higher at the low end and lower at the high end, but still in the same ascending order. Finally, the right hand side includes the possibility that $\Delta Aj = 0$, but this adds nothing to the value of the summation. The third inequality follows from the fact that the expression on the right hand side of the inequality is an upper bound on total cost increases because the marginal abatement cost function is positively sloped and the marginal cost at the new abatement level for those firms with cost savings represents a lower bound on cost savings per unit of increased emissions for each firm.

This proves the result for s € (0,1) when $t_e j$ are unique. When there at least two distinct permit taxes, it is possible to show that $MC^{10} < MC^{1N} < ... < MC^{mN} <$

MC^{mo} through slight modifications to the identical proof.

To show the result holds when s = 1, note that second inequality in (A7) becomes an equality, since the new marginal costs are now equal to each other, but the cost savings still exceed cost increases.■

If production targets are selected prior to pollution control decisions, then it can be shown that under the same conditions defined in the lemma, aggregate costs of pollution control decrease. Moreover, the lemma extends naturally to the case of multiple outputs and multiple pollutants with a single market in emissions, since all that is required is that the derived demand for the pollutant be downward sloping.

Theorem 3: Suppose firm j's tax structure is given by r_j - (t, t_1, ..., t_n) and $t_e j$, where r_i is a scalar and $t = t_1 = ... = t_n$. Let $t_e j' = 1 - [(1 - t_e j) / (1 - r_i t)]$. If $t_e j'$ are equal across all firms, then the solution is achieved. If tj are not equal across all firms, then the transformation

$t_e j^N = t_e^j, + s (K - t_e j')$, s €(0,1] and K € [0,1),

results in an increase in total firm profits measured at N th best prices.

Proof: Redefining $t_e j' = 1 - [(1 - t_e j) / (1 - r_i t)]$ means that the solution is now evaluated at the N th best prices. If tj are equal across all firms, then Theorem 1 applies. If $t_e^j \neq t_e i$ for some i,j, then Lemma 1 applies.■

Theorem 3 also can be extended along the same lines as Lemma 1.

FOOTNOTES

[1] Firms for whom output is fixed, or determined exogenously, such as utilities, can be modeled as cost minimizers within this framework. The results developed below apply to both kinds of firms.

[2] An additional term would need to be added if firms received an initial endowment of permits. Including such an endowment would not affect the subsequent results (see, e.g., Montgomery, 1972).

[3] Summations are presumed to be taken from i equal 1 to n unless otherwise indicated.

[4] The tax structure is generally represented by the vector $t, t_1, ..., t_n, t_e$.

[5] Formal proofs of the theorems are provided in the appendix.

[6] Interestingly, the taxes on these chemicals may now be binding.

[7] The shape of the E function follows from the assumptions regarding the production function.

[8] 1t may or may not be desirable to increase firm profits, per se. Reducing overall control costs is usually considered to be desirable.

[9] 1t is by no means obvious that losses and gains should be treated symmetrically in the permit market when the essence of the "regulatory bargain" with utilities is the asymmetric treatment of losses and gains in other activities. This issue deserves further study.

[10] Inputs are assumed to be non-negative. In the interest of simplicity, these constraints are assumed to be non-binding.

[11] I am indebted to George Jennings and Dan Ocone for suggesting a general approach to this proof. George Jennings derived the key parts of the proof involving the implicit function theorem and the inverse function theorem. His elegant contribution is gratefully acknowledged.

[12] The sign of $\partial G / \partial y < 0$ is the same as $\partial E / \partial y^j$), where y^j denotes the amount of input i used by firm j. This follows from setting up the appropriate Lagrangian and applying the envelope theorem (Varian, 1978, pp. 267-269).

[13] Analogous results for Theorem 2 and Corollary 1 obtain for the case in which there is an aggregate limit on output, there is a tradable permit market in output, and there are

individual firm quantity constraints on emissions. The proofs are virtually identical to the ones used here.

[14] I use the term costs instead of foregone profits for ease of exposition.