DOUBLE DIVIDEND RECONSIDERED

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ABSTRACT

A recent literature on optimal environmental taxation has rejected both the Pigouvian Principle and the ‘double dividend hypothesis’ in a second-best setting, claiming to have discovered a new distortionary economic phenomenon or ‘tax interaction effect.’ The current analysis finds that these results are actually due to comparing and combining monetary values expressed in incompatible units. When properly normalized these results do not contradicts either the Pigouvian Principle or double dividend hypothesis and there is no evidence of a ‘tax interaction’ effect.

When properly normalized, both analytical and numerical estimates indicate that the optimal pollution tax should be set above marginal environmental damages based on estimates for the US economy. Indeed, two different analytical models produce the identical result that pollution taxes should be set 33% above the Pigouvian rate. These findings support the claim that the welfare gains from an equal-yield environmental tax reform will be higher than what a Pigouvian analysis would suggest.

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I. Introduction

Recently a new literature on environmental taxation has appeared that calls into question many long-held beliefs about optimal environmental taxation. Initially the debate centered on the validity of the “double dividend hypothesis”—the notion that a revenue-neutral environmental tax reform which substitutes pollution taxes for other revenue-raising taxes can improve environmental quality and reduce the overall cost of tax distortions. This new literature rejects the double dividend hypothesis, but its implications are far broader and more profound. These analyses reject the “double dividend hypothesis” not because its basic intuition is wrong but, they argue, because of a previously unrecognized distortionary phenomenon, or “tax interaction effect” which produces potentially large distortions in a second-best world. Thus, the most surprising finding of this new literature is not the rejection of the double dividend hypothesis, rather it is the suggestion that the Pigouvian Principle must be modified in a second-best setting (see Bovenberg and de Mooij 1994; Bovenberg and van der Ploeg 1994; Parry 1995; Bovenberg and Goulder 1996; Fullerton 1997; Bovenberg and de Mooij 1997; Parry, Williams, Goulder 1999; and Goulder, Parry, Williams and Burtraw 1999).

The recent literature accepts the rational and intuition for a double dividend “in the sense that a cost reduction can be achieved by using revenues from pollution taxes to cut distortionary taxes rather than returning these revenues in a lump-sum fashion” (Bovenberg and de Mooij 1994, p. 1089). Nevertheless, this literature concludes that when a Pigouvian policy is introduced—where revenues are returned lump-sum to the economy—the optimal pollution tax will be lower than marginal environmental damages or even negative in some cases (Bovenberg and Goulder 1996). Given this result for a “Pigouvian policy”, the gains from using the pollution tax revenues to cut distortionary taxes will raise the optimal pollution tax, but will still generally
lie below marginal environmental damages (Bovenberg and de Mooij 1994; Bovenberg and van der Ploeg 1994; Parry 1995).

Based on these surprising results, the authors of this literature interpret them as evidence that revenue-motivated taxes actually exacerbate the distortionary effects of pollution taxes, causing the optimal pollution tax to lie below the rate which fully internalizes the marginal social damage from pollution. They argue that the marginal cost of environmental policy rises with the marginal cost of public funds (MCPF) and that the collective good of environmental quality directly competes with provision of other collective goods (Bovenberg and de Mooij, p. 1085). In arguing that a previously unrecognized distortionary phenomenon has been discovered, the authors of this literature recognize that this finding has far-reaching implications beyond analysis of environmental issues (Parry and Oates 1998).

Evidence supporting these claims comes from three distinct sources. First, several analytical models appear to demonstrate that the optimal pollution tax will lie below marginal environmental damages and will decline with a rise in the marginal cost of public funds (Bovenberg and de Mooij 1994, Bovenberg and van der Ploeg 1994; Bovenberg and Goulder 1996). Second, they claim that Sandmo’s (1975) seminal article corroborates their findings (Fullerton 1997; Bovenberg and de Mooij 1997; Schöb 1997; Bovenberg 1997). And third, results from numerical models also appear to support these claims (Bovenberg and Goulder 1996; Parry 1995; Parry, Williams, Goulder 1999).

Despite this substantial body of work, the current analysis finds that both the Pigouvian Principle and the double dividend hypothesis have been mistakenly rejected, and that these authors’ conclusions about optimal pollution taxes are not valid. In general these analyses involve normalization errors in which comparisons are made between monetary values that
reflect different and incompatible units. Specifically, an optimal pollution tax that is paid out of households’ net income is compared or combined with marginal environmental damages (numerical models) or the marginal cost of public funds (analytical models) which are measured in units of gross income.

When these results are re-normalized and expressed in comparable units, they do not contradict either the Pigouvian Principle or the double dividend hypothesis. Indeed, corrected estimates for the US economy derived from the most general analytical model in the recent literature (Bovenberg and Goulder 1996), produce results identical to those found using Sandmo’s optimal tax formula: pollution taxes should be set 33% above the Pigouvian rate.

The rest of this paper is organized as follows. A standard model is presented in section II, followed by an assessment of the implications of re-normalizing the tax rule in section III. Analytical models are reexamined in sections IV and V followed by a reappraisal of numerical models in section VI. The double dividend hypothesis is reexamined directly in section VI, and section VIII concludes.

II. A standard model

The issue is one of combining two, well-know optimal tax problems: environmental taxation and revenue-motivated taxation. We can state the problem formally as one in which \( m \) identical individuals who will maximize utility \( U = u(X_1, X_2, \ldots, X_k, Z, V, E, G) \) subject to the budget constraint, \( \Sigma_k X + Z = (H-V) \). There are \( k \) non-polluting \( X \) goods and one good \( Z \) which erodes environmental quality, \( E \), such that \( E = e(mZ) \) and where \( \frac{de}{d(mZ)} < 0 \).

Units are chosen for both goods and for labor so that all pre-tax prices equal one. Leisure \( V \) is taken out of the time endowment, \( H \), so that labor supply \( L = (H - V) \). We will assume that
production has a simple linear technology where \( mL = \Sigma mX + mZ + G \). Government supplies an exogenous amount of public goods, \( G \), by collecting revenue \( T \), where we assume \( G = T \).

Individuals will maximize utility subject to their budget constraint while ignoring the environmental consequences of their own consumption of the polluting good, and they will take \( G \) as given. Writing this problem as a Lagrangian equation we have

\[
\prime = u\left( \sum_k X, Z, V, E, G \right) + \lambda \left( H - V \right) - \sum_k \left( 1 + t_x \right) X_j - \left( 1 + t_z \right) Z
\]

In turn, government will solve the optimal tax problem for a world of \( m \) identical individuals represented by the Lagrangian equation

\[
\prime = mu \left( \sum_k X_j^*, Z^*, V^*, e(mZ^*), G \right) + \mu \left[ T - \sum_k mt_x X_j^* - mt_z Z^* \right]
\]

where the \( X^*, Z^*, \) and \( V^* \) are the solutions to the individual’s choice functions.

Government may introduce taxes on the \( X_j \)s and \( Z \) both to raise revenue and to correct the environmental externality. Thus we can now state the overall optimal tax problem as

\[
\max_{m} m \left\{ \max_{X, Z, V, E, G} u \left( \sum_k X, Z, V, E, G \right) + \lambda \left( H - V \right) - \sum_k \left( 1 + t_x \right) X_j - \left( 1 + t_z \right) Z \right\}.
\]

The general solution to this problem was provided by Sandmo (1975). His formula for the optimal tax on \( Z \), and its implications are examined in detail below.

In the recent literature and in the current analysis our interest is on the portion of the optimal tax on \( Z \) attributable to pollution. At issue is whether the pollution tax (component) on \( Z \)
is greater than or less than marginal environmental damages (MED), where MED is the Pigouvian rate and is defined from the first-order relationships as

$$\text{MED} = -\frac{m \frac{\partial u}{\partial E} \frac{\partial e}{\partial (mZ)}}{\lambda}.$$  [4]

where the numerator on the right-hand side is the marginal social damage from the pollution caused by consuming $Z$ in utility terms, and this is divided by the marginal utility of income, $\lambda$. It is worth emphasizing that the optimal tax on $Z$ will include both a revenue-motivated, or Ramsey, component and a corrective, pollution component. Thus, it is quite possible that the total tax may exceed MED even if the pollution component of the tax is less than MED (see Fullerton 1997, and Schöb 1997 on this point).

For a given total optimal tax, the portion attributable to pollution is not obvious. In reality, Ramsey taxes will vary among goods so that the differential between the optimal tax on a polluting good and a non-polluting good may reflect differences in their Ramsey taxes as well as the pollution tax. A convenient way to isolate the environmental portion of the optimal tax on $Z$ is to assume that the Ramsey taxes on all goods, $X_s$ and $Z$, are equal. The practice in the recent literature has been to assume certain restrictions on preferences which ensure equal Ramsey taxes. This has been accomplished by assuming that utility is homothetic in consumption goods while weakly separable in leisure, environmental quality, and government consumption (Bovenberg and de Mooij 1994; Bovenberg and Goulder 1996).\(^1\)

\(^1\) The differential between the optimal taxes on polluting and non-polluting goods is an empirical question, and thus we can draw no precise conclusions here applicable to any specific case. The recent literature, and the current analysis, can be interpreted as asking what is true for an average, or typical, good. For the purpose of answering that question, the assumptions here are judged to be both reasonable and sufficient.
These restrictions allow us to assume that the optimal revenue-motivated Ramsey tax, \( t^R \), will be the same on all goods. Knowing that the optimal tax on \( Z \) is \( t_Z = t^R + t^P \) where \( t^P \) is the pollution component, then we can infer that \( t^P = t_Z - t_X \) where \( t_X = t^R \). The simplifying assumptions regarding preferences notwithstanding, this approach provides a sound and judicious way to compare optimal pollution taxation in a second-best setting with the Pigouvian Principle with either analytical or numerical models, and to assess how the double dividend hypothesis will affect optimal environmental taxes.

III. Tax normalization

Each contribution to the recent ‘tax interaction’ literature differs from the standard model presented above in one critical way. In every case the household budget constraint in [3] has been re-normalized to reflect a tax rule where an income tax is the primary revenue-raising tax, and thus where the relevant monetary unit is net, or after-tax, income rather than gross income. The implications of this modification for the analyses and comparisons undertaken was not recognized, and this is the source of the mistaken interpretations of their results. Before reexamining the specific results in the recent literature, this section seeks to clarify exactly how this re-normalization affects the key monetary values in the model.

In general, we understand a normalization to be any operation which multiplies all elements of an equation by a scalar, such as a conversion of physical units of measure, or converting from one currency to another. And we also understand that a re-normalization will not effect an equation’s equilibrium values or outcomes. In the current context, the re-normalization in question is interpreted as, and intended to represent, a change in the mix of
expenditure and income taxes, but algebraically it is identical to one which converts units from US dollars to UK pounds.\textsuperscript{2}

The recent literature re-normalizes a portion of the model in [3] by multiplying the budget constraint by $\beta = (1 - t_L)$, where $t_L$ is the income tax rate, so that we have

$$
\max \left\{ m \left( \max_{X, Z, V} \left\{ u \left( \sum_k X_j, Z, V, E, G \right) \right\} \right) \right. \\
+ \lambda \left[ \beta (H - V) - \sum_k \beta (1 + t_X) X_j - \beta (1 + t_Z) Z \right] \\
+ \mu (T - mt_X X_j - mt_Z Z) \right\}
$$

This particular re-normalization represents a convenient special case if we assume further that $\beta = (1 - t_L) = 1/(1+t^R)$. Given that an income tax is equivalent to a uniform tax $t_X$ on all goods where $(1+t_X) = 1/(1 - t_L)$, we can see that, given the restrictions on preferences introduced above, the income tax is identical to a uniform Ramsey tax on all goods. As a result of this, a zero tax will be optimal on the $X_j$s, and the only expenditure tax will be the pollution component of the tax on $Z$. Thus we can write the problem as

$$
\max \left\{ m \left( \max_{X, Z, V} \left\{ u \left( \sum_n X_j, Z, V, E, G \right) \right\} \right) \right. \\
+ \lambda \left[ (1 - t_L) (H - V) - \sum_n X_j - (1 + t^P) Z \right] \\
+ \mu (T - mt_X X_j - mt_Z Z) \right\}
$$

\textsuperscript{2} Whether converting between alternative physical units of measure, or from one currency to another, the normalization of an algebraic relation is understood to produce a new relation equivalent to the original one. A normalization should not alter the underlying relationships or equilibrium outcomes in the equation because the correspondence between the elements in the relation is maintained. This is true because a normalization can be interpreted as multiplying by one. An equation $f(x) = g(x)$ can be written as $(f(x)/g(x)) = 1$. And we can multiply the left-hand side of this by one where $1 = \beta/\beta$. This will give us $(\beta f(x)/\beta g(x)) = (f(x)/g(x)) = 1$ which we can write as $\beta f(x) = \beta g(x)$, and we can be sure that this will not alter the equation. We may also normalize a single term in an equation, but only if we multiply the term
Thus, this normalization simplifies the task of isolating the pollution component of the optimal tax on Z since the entire tax on Z will be the pollution tax, \( t_Z = t^P \).

The re-normalization above, however, is a partial one and it complicates the analysis in several critical ways. First, the optimal pollution tax in [6] will differ from [3]. Second, the value and interpretation of \( \lambda \), the Lagrange multiplier, will differ since it will reflect the marginal utility of a unit of net rather than gross income. Third, since \( \lambda \) will be altered, MED will also differ with the re-normalization given the definition of MED in [4]. The precise effect on each of these is evaluated in the remainder of this section.

A. optimal pollution tax. The effect of re-normalizing the household budget constraint on the optimal pollution tax is straightforward. For any equilibrium outcome in the standard model [3], we obtain a set of optimal Ramsey taxes \( t^R \) on the Xs, and an optimal tax \( t_Z \) on Z. The budget constraint is written as \((1 + t^R)X + (1 + t_Z)Z = (H-V)\) and the optimal pollution tax (component) will be the difference between the two, \( t^P = t_Z^* - t^R^* \). Re-normalizing this budget constraint by any scalar \( \beta \) will produce an equivalent result,

\[
(\beta + \beta t^R)X + (\beta + \beta t_Z)Z = \beta(H-V).
\]

Here we can see directly that the differential between the optimal tax on Z and X is now \( \beta t_Z^* - \beta t^R^* = \beta t^P \) rather than \( t^P \).

If we further define the scalar \( \beta = (1 - t_L) \) where \( (1 - t_L) = 1/(1 + t^R) \), we can write the budget constraint as \( X + (1 + (1 - t_L)t^P)Z = (1 - t_L)(H-V) \), in which case the optimal tax on X is now zero, and the pollution tax is \( (1-t_L)t^P \), where \( t^P \) was the optimal pollution tax under the initial normalization. In either case, the optimal pollution tax is reduced in proportion to the

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by one. For example, if we have \( y = f(x) + g(x) \), we can multiply \( f(x) \) by \( \beta/\beta \) without altering the relationship, or rewriting the original equation as \( y = \beta(1/\beta)f(x) + g(x) = f(x) + g(x) \).
normalization scalar $\beta$. However, this is only the result of a change in normalization and cannot have any real effect on allocation, welfare, or our test of the double dividend hypothesis.

B. Marginal environmental damages. To evaluate how this re-normalization affects MED, consider two equivalent models with different normalizations, one with an income tax equivalent to a uniform Ramsey tax, and one where the income tax is zero. The household optimization problem for [1] above becomes

$$\dot{\prime} = u(X, Z, V, E, G) + \lambda^G (H - V) - (1 + t^R)X - (1 + t_Z)Z$$  \[7\]

using the superscript ‘G’ for the gross income units of the budget constraint. Using the notation $U_J = \frac{\partial u}{\partial J}$, we obtain the first-order condition, $U_X - \lambda^G (1 + t^R) = 0$.

Under an equivalent normalization where an income tax $t_L$, collects the same revenues so that $(1 - t_L) = 1/(1 + t^R)$, and using the notational superscript, $N$ for net income, the maximization problem can be written as

$$\dot{\prime} = u(X, Z, V, E, G) + \lambda^N (H - V)(1 - t_L) - X - Z(1 + t^R)$$  \[8\]

with first-order condition $U_X - \lambda^N = 0$.

Since these two equilibria imply identical allocations, we know that $U_X$, $U_E$, and $U_V$ are the same in both. Combining their respective first-order conditions, can write

$$\frac{\lambda^N}{(1 + t^R)} = \lambda^G = \lambda^N (1 - t_L)$$  \[9\]

demonstrating that the marginal utility of gross income will decline relative to the marginal utility of net income in proportion to the income tax rate.

Given MED as defined in [4], the effect of the re-normalization on MED will be that the net income measure of environmental damages, $MED^N$, will decline relative to our gross income measure, $MED^G$, in proportion to the income tax, or $MED^N = (1 - t_L)MED^G$. 


Thus we find that with this re-normalization, the optimal pollution tax and MED should both be reduced in proportion to the income tax rate. As a result, the re-normalization should have no real effect on our comparison of optimal pollution taxes and MED. However, the change in normalization implies not only an entirely different tax system, but, importantly, it redefines the monetary units in the model.

As a practical matter, our measure of MED is lowered in a world with an income tax, but this will already be reflected in our valuation of the environment since both environmental taxes and individuals’ valuation of environmental damage will be evaluated in net income units.

To be clear, choose good \( X \) as numeraire and compare the first-order conditions for the Ramsey tax program and the equivalent income tax program above. Assume that at equilibrium the marginal rate of substitution between a unit of \( X \) and a unit of environmental improvement (-MED) is \( \phi \). We can express the willingness to pay for environmental improvement with a gross income normalization (\( WTP_E^G \)) as

\[
WTP_E^G = \phi \frac{U_X}{\lambda} = \phi (1 + t^R)
\]  

[10]

And we can write willingness to pay for environmental improvement for a net income normalization (\( WTP_E^N \)) as

\[
WTP_E^N = \phi \frac{U_X}{\lambda} = \phi
\]  

[11]

Given that \( U_X, U_E, U_V \), are equal for both normalizations, we can combine [10] and [11] to write

\[
WTP_E^N = \frac{WTP_E^G}{(1 + t^R)}
\]  

[12]

The measured value of MED will differ because the two models are normalized differently, employing different definitions of the monetary unit of exchange. In the real world,
revealed-preference or stated-preference valuation techniques will generally reflect choices by individuals (or their willingness to pay) out of their net income, and these measures will therefore already be appropriately normalized.

C. Re-normalization and the cost of public funds. In addition to the above effects of the re-normalization, it is noteworthy that the units of $\lambda$ and $\mu$ associated with the two constraints in our problem are no longer equivalent. We have re-normalized the household budget constraint to units of net income but we have left the units in the government budget constraint unchanged.

Comparing the standard normalization in [3] with the re-normalized model in [5] or [6] we see that while the household budget constraint has been re-normalized, the revenue constraint has been left unchanged. The standard normalization has both private income and public funds expressed in common units of gross income (or we can take a unit of leisure as the numeraire). By contrast, in the re-normalized model the units of the household budget constraint correspond to net income, a fraction $(1-t_L)$ of gross income, and therefore a similar fraction of a unit of leisure. However, since the revenue constraint remains normalized in units of gross income, $\mu$ will reflect the marginal value/cost of revenues measured in gross income units.

One way of avoiding this inconsistency in units would be to re-normalize the entire optimization problem including both constraints. From [5], we can simply multiply the second constraints by $\beta$ so that all monetary elements of the optimization problem have been similarly normalized. As with $\lambda$, we can show that $\mu^G = \beta \mu^N$.

To summarize, by including an income tax as part of the optimal tax program in their models, the recent literature has introduced several complications with implications that were not fully appreciated. The presence of an income tax in the household budget constraint involves the redefinition of the monetary unit in the model. And as a result of this it alters our measure of the
marginal utility of income, marginal environmental damages, and the optimal pollution tax. Moreover, because only the household budget constraint has been re-normalized, it creates an incompatibility between the units in the household budget constraint and the government’s revenue constraint.

IV. Analytical models.

General analytical results in the recent ‘tax interaction’ literature are derived in Bovenberg and Goulder (1996), and also in Bovenberg and van der Ploeg (1994). Considering only the results for polluting goods which cause pollution (as opposed to inputs), the essential elements of their model can be characterized by [6]. The only difference between this model and the standard problem solved by Sandmo (1975) is the re-normalization in the recent literature. If we introduce the restrictions on preferences that ensure equal Ramsey taxes for all goods, then these two models represent different normalizations of the same basic problem, and they should therefore produce identical results for given parameters.

In the Bovenberg and Goulder model, optimal tax rates are derived in the usual fashion, solving the government’s problems of maximizing household utility subject to the government budget constraint and given the maximizing behavior of firms and households (see Bovenberg and Goulder 1996, Appendix A). The solution for the optimal tax on the ‘dirty consumption good’—using current notation—is found to be

$$t^p = \left( m \frac{\partial u}{\partial E} \frac{\partial e}{\partial(mZ)} \right) \frac{\lambda}{\mu} \cdot \left[13\right]$$
The interpretation provided by Bovenberg and Goulder is that, since the term in brackets equals MED and $\mu$ is greater than one, we can see by inspection that the optimal pollution tax $t^p$ will be less than MED, and that $t^p$ will decline with a rise in the marginal cost of public funds. They interpret this finding by suggesting that “the presence of distortionary taxes requires a modification of the Pigouvian principle. … The higher the MCPF, the greater the cost of public consumption goods, including the public good of environmental quality. When these goods are more costly, the government finds it optimal to cut down on public consumption of the environment by reducing the pollution tax.”(1996, p. 987). For their model of the US economy, they estimate the income tax rate to be 40% and the marginal cost of public funds to be 1.25. These parameters produces an optimal pollution tax of 0.80-MED.

The expression they have derived, however, involves different and incompatible units and cannot be interpreted in the way they have suggested. As explained above, their model is one in which the household budget constraint and $\lambda$ are expressed in units of net income while the government revenue constraint and $\mu$ reflect gross income units. As a result of this, the units on the right-hand side are not equivalent to those on the left, and the expression cannot be interpreted directly. To see this clearly, we can write out the units in [13] using the notation $U$ for marginal utility, $Y^G$ for a unit of gross income where $Y^G = (H-V)$ and $Y^N = (1-t_L)(H-V)$ for a unit of net income, to get

$$\frac{Y^N}{Z} = \left( \frac{U}{Z} \right) \frac{U}{Y^N} \frac{U}{Y^G} \frac{Y^G}{Z} = \frac{Y^G}{Z}$$

3 Bovenberg and Boulder replace $\lambda/\mu$ with $1/\eta$ where they have defined $\eta \equiv (\mu/\lambda)$. They further define $\lambda$ in the Appendix in terms of the non-polluting good, or using current notation, $\lambda \equiv \partial U / \partial X$. 

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When simplified, we see that the units on the right-hand side are gross income units per unit of Z whereas the tax rate we want is one corresponding to net income units.

This oversight can be easily corrected by converting to compatible units. Given that $\mu^G = \beta \mu^N$, we can substitute in [13] to produce the expression

$$
t^p = \left( \frac{m \frac{\partial u}{\partial E} \frac{\partial e}{\partial (mZ)}}{\lambda} \right) \frac{\lambda}{(1 - t_L) \mu^N}
$$

where $\mu^N$ is the marginal cost of public funds per unit of net income. With this correction, we can now substitute Bovenberg and Goulder’s base case parameters\(^4\) to get

$$
t^p = \left( \frac{m \frac{\partial u}{\partial E} \frac{\partial e}{\partial (mZ)}}{\lambda} \right) \frac{1}{(1.25 * 0.6)} = 1.33 \text{MED.}
$$

In contrast to the Bovenberg and Goulder interpretation that their expression implies an optimal pollution tax less than MED, when the expression is represented in compatible units, these same parameters produce an optimal pollution tax 33 percent above MED.\(^5\)

\(^4\) Here $\mu$ is the cost of a unit of public funds expressed in utility terms per unit of revenue expressed in net income units. Our empirical estimates of the MCPF, like those used by Bovenberg and Goulder, are unitless ratios where the utility measure of $\mu (=U/Y^G)$ has been monetized, dividing the numerator by the marginal utility of gross income, or MCPF = $((U/(U/Y^G))/Y^G = Y^G/Y^G)$. Thus, an estimate of MCPF = $1.25$ is a unitless ratio unaffected by the substitution of $\mu^N$ for $\mu^G$.

\(^5\) Bovenberg and de Mooij’s (1994) “two-good” model also employs a non-standard normalization which complicates the interpretation of their result (they explore the welfare effects of a revenue-neutral tax policy experiment in which the monetary unit, and hence MED, are a function of the tax. Nevertheless, Fullerton (1997) and Metcalf (1999) find that even with a standard normalization this model produces an optimal pollution tax less than the Pigouvian rate. As explained in Appendix A, however, these analyses are specific to a ‘two-good world’ and they contradict the results from more general models.
V. Sandmo’s optimal tax formula

How do the results above correspond to results based on the optimal tax formula from Sandmo’s (1975) seminal article? Fullerton (1997), Schöb (1997), Bovenberg and de Mooij (1997) and Bovenberg (1997) cite Sandmo’s result to support their claim that the optimal pollution tax should be less than marginal environmental damages, closer examination reveals just the opposite. They express Sandmo’s formula as

\[ \theta = \left(1 - \frac{1}{\mu}\right)R + \frac{1}{\mu} \tau \]  \hspace{1cm} [15]

where \( \theta \) is the optimal tax rate, \( R \) is the Ramsey term, \( \mu \) is the marginal cost of public funds, and \( \tau \) is the Pigouvian rate. Pointing out that the second term on the right-hand side will be zero for non-polluting goods, these authors suggest that the differential between the optimal tax on a polluting good versus a similar non-polluting good will simply equal the value of this second term—since the first term will be the tax on non-polluting goods. Since this second term is just equal to the Pigouvian rate divided by \( \mu \), and since \( \mu \) is assumed to be greater than one, they conclude that “the difference between the tax on the dirty good and the tax on the clean good…is less than the Pigouvian rate” (Fullerton 1997, p. 250).

Closer examination, however, reveals that this interpretation mistakenly assumes that the denominator of \( \theta \) is exogenous. In Sandmo’s formula, however, both \( \theta \) and \( \tau \) are tax-inclusive \textit{ad valorem} rates such that \( \theta = \frac{t}{p} \) and \( \tau = \frac{MED}{p} \), where \( p = q + t \) with \( q \) is the pretax price of the good. In detail, Sandmo’s tax rule is

\[ \frac{t}{q + t} = \left(1 - \frac{1}{\mu}\right)R + \frac{1}{\mu} \frac{MED}{q + t} \]  \hspace{1cm} [16]
Sandmo’s formula represents two equations that must be solved simultaneously; the second one being the identity \( p = q + t \). Consequently, the casual interpretation above would be correct only if the denominator on the left-hand side was the pre-tax price, \( q \).

For current purposes, we want to evaluate the difference between the unit tax on a polluting good \( t_Z \) and that on a non-polluting good \( t_X \) where our restrictions on preferences imply \( R \) in Sandmo’s model will be the same for all goods. We can write the optimal taxes for \( Z \) and \( X \) respectively as

\[
t_Z = \left(1 - \frac{1}{\mu}\right)R(q_Z + t_Z) + \frac{1}{\mu} MED \\
= \left(1 - \frac{1}{\mu}\right)R(q_Z + t_Z) + \frac{1}{\mu} MED
\]

\[
t_X = \left(1 - \frac{1}{\mu}\right)R(q_X + t_X)
\]

We can write the tax differential \( t_Z - t_X \) as

\[
t_Z - t_X = \left(1 - \frac{1}{\mu}\right)R(q_Z + t_Z) + \frac{1}{\mu} MED - \left(1 - \frac{1}{\mu}\right)R(q_X + t_X).
\]

Assuming equal pre-tax prices, \( q_X = q_Z \), we can simplify this as

\[
t_Z - t_X = \left(1 - \frac{1}{\mu}\right)R(t_Z - t_X) + \frac{1}{\mu} MED
\]

which can be further simplified as

\[
t_Z - t_X = \frac{MED}{(\mu - \mu R - R)}
\]

We see that, given \( \mu > 1 \), the denominator on the right-hand side will be less than one, and hence the optimal pollution tax will exceed MED, so long as \( R > 1 \).

From our standard optimal tax rules we can infer a value for \( R \) based on the tax rate and MCPF. Indeed, Sandmo’s formula in [16] reduces to this standard optimal tax rule in the absence of an externality, which we can be rearranged as
Once again using Bovenberg and Goulder’s parameters for the US economy (μ = 1.25 and t_L = 0.40), we need an intermediate step to convert the income tax of 0.40 into its equivalent uniform commodity tax, or t_X = 0.667 since (1+t^R) = 1/(1-t_L). Given this value we can compute the value R = 2. Now, solving [16] gives us

\[ t_Z - t_X = \frac{MED}{(1.25 - 1.25 \times 2 - 2)} = 1.33MED \]

Once again we have the result that the optimal pollution tax should exceed MED.

Moreover we have produced a result identical to the one we estimated from the Bovenberg and Goulder model using the same values for the tax rate and MCPF. Indeed, for any given values for the tax rate and MCPF, the Sandmo model and the (correctly normalized) Bovenberg and Goulder model will produce identical estimates for the optimal pollution tax because they represent different normalizations of essentially the same model.

When the parameters in each case reported in Bovenberg and Goulder’s sensitivity analysis are applied to this derivative of Sandmo’s formula, the results are identical to Bovenberg and Goulder’s (compare the last two columns in table 1). Importantly the first three of the results presented in table 1 indicate that with a rise in the marginal cost of public funds, the optimal tax on a polluting good should rise by more, not less, than the optimal tax on a non-polluting good. This result contrasts with the suggestion by Fullerton (1997) and Schöb (1997) that the results from the ‘tax interaction’ literature support the ‘modified Samuelson Rule’ in which the optimal provision of a public good declines with a rise in the marginal cost of public
funds. These results indicate that the optimal tax on a pollution-causing good will rise faster than the optimal tax on a non-polluting good.

VI. Numerical models

The numerical models in the ‘tax interaction’ literature (Goulder 1995; Bovenberg and Goulder 1996; Goulder, Parry and Burtraw, 1997; Parry, Williams and Goulder 1999) have produced estimates consistent with their corresponding analytical results, indicating that “in the presence of distortionary taxes, optimal environmental tax rates are generally below the rates suggested by the Pigouvian principle—even when revenues from environmental taxes are used to cut distortionary taxes” (Bovenberg and Goulder, 1996, p. 994). It seems unlikely, however, that the kinds of incompatible normalizations identified above would arise in computable general equilibrium models, since the optimal values for taxes and $\lambda$ are determined endogenously in these models based on whatever normalization has been introduced. Indeed, if the model being solved were precisely the one described in [6], then values for both MED and the pollution tax would correspond automatically to units of net income.

However, the common practice in the ‘tax interaction’ literature has been to simplify the model for convenience, introducing government behavior as an additional constraint on a single household optimization problem rather than as two separate and independent “nested” optimization algorithms for households and government. In addition to this, these models have adopted the practice of returning revenues $T$ to individuals as a lump-sum government transfer into their household budget. This has the advantage of preserving real income levels for policy
experiments such as raising or lowering tax rates, etc., and it is desirable in some general
equilibrium models for accounting purposes and to ensure a general equilibrium solution.\footnote{Based on personal correspondence with Roberton Williams III.}

Unfortunately, these particular practices alter the correspondence between the intended
normalization of the model and the values produced endogenously for $\lambda$. The models actually
being estimated tend to take a form that can be characterized as

\[
\dot{\lambda}(H - V)(1 - t_L) + R - \sum X - (1 + t_Z)Z] + \mu(t_L(H - V) + t_ZZ - T) \tag{19}
\]

where revenue $T$ is fixed exogenously.\footnote{Of course households must ignore this rebate in making consumption decisions, otherwise they would simply ignore all taxes. This condition can be ensured, for example, by introducing the household’s first-order conditions as constraints. This is the approach used in the model in the Appendix.} This formulation blurs the distinction between gross and net measures of the marginal utility of income and therefore create ambiguity about the proper measure of MED\cite{Parry1995; Parry, Williams and Goulder1999}. Optimizing such a model with the revenue constraint binding, we can substitute $T = t_L(H - V) + t_ZZ$ into the budget constraint as

\[(H - V)(1 - t_L) + t_L(H - V) + t_ZZ = X + (1 + t_Z)Z\]

or gross income. Thus, this formulation is equivalent to one where the constraint is on gross
income, and yet the tax being imposed on pollution will be paid out of net income by individuals.
As a result, the endogenously determined value of $\lambda$ will represent the marginal utility of a unit
of gross income rather than net income.

An alternative formulation of the numerical models produces a public good $G$ which enters as a separable argument of the utility function, rather than rebating revenues $T$ \cite{Bovenberg and Goulder1996}. The implications of this formulation will be the same, however. This model can be written as

\[u(X, Z, V, E, G) + \lambda[(H - V)(1 - t_L) - X - (1 + t_Z)Z] + \mu(t_L(H - V) + t_ZZ - T]}

\]
where \( T=G \), and where the marginal utility for an increment in net income, \( d((H-V)(1-tL)) \), will correspond to marginal utility from consumption \( (U_XdX + U_ZdZ) \). Each unit of \( (1-tL)(H-V) \) will also require an increase in revenues \( d(H-V)tL \) and a rise in \( G \) producing marginal utility \( U_GdG \).

Of course the cost of \( dG \) will exceed that of private goods by \( 1/\mu \) given the distorting effect of taxation, but this will be offset at the optimum by a higher marginal utility of \( dG \) given the first-order conditions whereby \( U_{\Delta} = \left( \frac{\mu}{\lambda} \right) U_x \).

Thus, for reasons of computational convenience, these numerical models have inadvertently been designed so that the estimated value of \( \lambda \), and consequently MED, corresponds to gross income units, while the optimal taxes correspond to payments made out of net income.

Once again these incompatibilities can be reconciled by converting from the gross to net income measure of MED multiplying by \( (1-tL) \), and comparing this directly to the optimal pollution taxes. In the numerical results from the Bovenberg and Goulder model, \( MED^G \) is set equal to $75. With an income tax of 40 percent, this corresponds to \( MED^N = $45 \). In Bovenberg and Goulder’s ‘realistic tax system,’ their base case estimate for the optimal pollution tax is $52, or 16 percent above MED.

A similar reevaluation can be made for Parry’s (1994) analysis which uses a distinct graphical approach to estimate the welfare costs of an environmental tax reform evaluated based on elasticities for small changes in price, quantity, and tax. His numerical results are consistent with the others in the literature, and like those models all tax revenues are assumed to be given back lump-sum to households. This produces a model in which MED is assumed to be independent of the income tax, and effectively represents a gross income measure of MED, while at the same time households must pay pollution taxes out of their net income. Re-normalizing his results to units of gross income (or equivalently converting his measure of MED to a net income
measure), the optimal pollution tax for his base case is 10 percent higher than MED. That is, with an income tax of 0.43, we can compute the gross income tax on pollution as 0.63MED/(1-0.43) = 1.10MED.

There are several ways to avoid producing incompatible units in numerical models of this kind. First, the models can adopt a standard gross income normalization. Second, the revenues collected by government can simply be assumed to disappear. Or third, a more complicated algorithm could be used which solves individual’s and the government’s optimization problems independently.

Taking the first of these approaches, a simple numerical CES utility function corresponding to the assumptions maintained by Bovenberg and de Mooij (1994) is

\[ U^i = (X^{-v} + Z^{-v})^{-k/v} + aV^g + E. \]

The details of the full model are described in Appendix B. Simulation results for the optimal taxes corresponding to a range of revenue requirements were generated and presented as a share of MED in figure 1. These results indicate that once revenue requirements cannot be satisfied with pollution taxes alone, the optimal pollution tax (the differential between the optimal tax on Z and the optimal tax on X) is higher than MED and rises with rising revenue requirements. The results are consistent with both the Pigouvian Principle and the double dividend hypothesis.

VII. Double dividend reconsidered

The double dividend hypothesis refers, not to the question of whether the optimal pollution tax should exceed MED, but rather to the welfare effects of an equal-yield environmental tax reform that substitutes pollution taxes for pre-existing revenue-motivated taxes. There is indeed a correspondence between these two issues, but this has been confused by
the mistaken results in the ‘tax interaction’ literature. Additional confusion has been caused by multiple and misleading definitions of the double dividend hypothesis (‘strong’ versus ‘weak’ forms), and by the absence of any intuitive explanation for why we might expect a double dividend in the first place.

To clarify each of these sources of confusion it is advantageous to model pollution, or “environmental waste disposal services,” as a separate good, and thus to consider a tax on emissions directly. We can modify [3] so that the environmental resource in question (e.g., air, water, or atmosphere) is defined explicitly with an assimilative capacity \( Q \) to absorb and eliminate residual wastes \( W \). Environmental quality \( E \) is a function of \( Q \) and \( W \). We will assume here a simple static form of the relationship, \( Q = E + W \).

The level of emissions will vary positively with the consumption of goods which generate waste, and negatively with substitutes for those pollution-generating goods and also with abatement goods which reduce the amount of pollution per unit of consumption. Thus we expect \( W = \omega(\Sigma X_j) \) where some \( X_j \)s may be abatement good such that \( \frac{\partial W}{\partial X_j} < 0 \). We can therefore take a household production function approach (Michael and Becker, 1973) and assume all market goods are inputs used in household production processes of the non-market sector such that consumer demands for these goods are the derived demands which we introduce directly into the derived utility function. We can thus modify [3] by substituting as \( Q = E + W \) to write it as

\[
\max \left\{ \frac{\partial u}{\partial x} \left( \sum_{n=1}^{\infty} X_n, W, V, (Q - W), G \right) \right. \\
+ \lambda \left( \{ H - V \} - \sum_{n=1}^{\infty} (1 + t_X) X_n - t_w W \right) \\
+ \mu \left( T - m \sum_{n=1}^{\infty} t_X X_n - mt_w W \right) \}
\]  

[20]
A. Emissions taxes

With this modest modification in our model we can reexamine the relationship between optimal pollution taxes and MED. Treating environmental services like any other good we can apply Sandmo’s optimal tax expression directly to emissions, \( W \). Sandmo’s formula in [16] can be rearranged as

\[
t = \left( \frac{q - 1}{\mu} \right) R + \frac{MED}{\left( \mu - \mu R + R \right)}.
\]  

[21]

In the case of emissions, however, we have \( q = 0 \) so that the first term above drops out leaving the relation

\[
t = \frac{MED}{\left( \mu - \mu R + R \right)}.
\]  

[22]

This expression is comparable to the one above for the tax differential between polluting and non-polluting goods: the pollution tax will exceed MED whenever \( R > 1 \). If we maintain our restrictions on preferences so that equal Ramsey terms can be assumed for all goods including the environment\(^8\), we get the same result as before when assuming \( R = 2 \) and \( \mu = 1.25 \), or \( t^P = 1.33 \text{MED} \).

However, the intuition for why the pollution tax will generally exceed MED becomes more transparent in this model where environmental services are separate goods. In a first-best world we want to price all goods at their social cost, including environmental goods so \( t^P = \text{MED} \). In a second-best world the general result implies that we want to add a Ramsey tax premium to all goods, including environmental goods, raising their tax-inclusive price above social cost. Thus, the optimal tax on pollution will generally include a corrective or Pigouvian component as well as a revenue-motivated or Ramsey component. Of course, asymmetric

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\(^8\) This assumption is no more restrictive (or less realistic) than in the previous models. It explicitly separates the household’s direct objects of choice (primary goods) from the means used to produce them. For example, \( U = u(f(X_1, X_2), g(X_3, W)) \) where \( u, f, \) and \( g \) are homothetic functions would satisfy this condition.
assumptions such as being a stronger-than-average substitute for leisure, carry the possibility of a negative Ramsey tax, but this is true for any good including the environment.

The current formulation also helps settle one aspect of the double dividend debate. In defining the double dividend hypothesis and discussing its possible implications, Goulder (1995) asks whether a pollution tax might be justifiable even if it is uncertain whether marginal environmental damages are positive. The result above provides an unambiguous answer: if MED = 0 then the optimal pollution tax will also equal zero: taxing a good which has no social cost will not be optimal if it causes individuals to substitute toward goods which are costly to produce.

More generally, the relation in [22] indicates that the optimal pollution tax is proportional to MED for given values of µ and R. As a result, justification for a given size environmental tax requires more than knowing that MED is non-negative. Nevertheless, if MED were known to be positive but of uncertain magnitude, the results here provide some room for reducing the ‘burden of proof’ for policymakers. For example, if policymakers are confident that MED is at least $30 but uncertain whether it is as high as $40, then a revenue-neutral environmental ‘tax swap’ which introduces a pollution tax of $40 can be justified on the grounds of a $30 estimate of MED, since applying the Bovenberg and Goulder parameters to [21] gives the same result that the optimal pollution tax should be 33 percent above MED.

B. Environmental tax reform. If our starting point is a world with revenue-motivated taxes but without environmental taxes, what are the welfare implications of an equal-yield tax reform which raises pollution taxes and lowers other taxes? The most common interpretation of the double dividend hypothesis is that the welfare gains will exceed those anticipated by a Pigouvian analysis (where revenues are returned to the economy lump-sum). Although the precise definition of the double dividend hypothesis has recently been debated.

Goulder has introduced a distinction between a ‘strong’ versus ‘weak’ form of the double dividend hypothesis. In its so-called ‘strong form’ the substitution of environmental taxes for pre-existing taxes must have a positive welfare effect over some range, independent of the
environmental gains, implying a more efficient tax system in addition to greater environmental efficiency. Goulder and other contributors to the recent literature have rejected this ‘strong form’ of the double dividend hypothesis due primarily to the belief that their ‘tax interaction effect’ was a real distortionary phenomenon rather than a normalization error (Goulder 1995, Bovenberg and Goulder 1999 forthcoming; Parry, Williams and Goulder 1999).

With a model where environmental services are identified as separate goods, we have an intuitive way to see the context in which the ‘strong form’ of the double dividend hypothesis will be valid. Assume our starting point is a set of ‘optimal’ taxes for $W$ with the exception that $t_W = 0$. Our second-best problem has been reduced to one in which there is only one source of inefficiency: good $W$ is untaxed. Clearly the introduction of a tax on $W$ will represent a broadening of the tax base, and, over some range, it should reduce the excess burden of the tax system overall.

Given the well-established notion that the marginal excess burden of a tax rises (more than proportionally) with the tax rate, we can assume that for any good, but in this case for $W$, that the $\text{MCPF}_W \ll \text{MCPF}_X$ when $t_W \ll t_X$. From our starting point, then, the introduction of a small tax $\Delta t_W$ will incur a smaller cost per dollar of revenue than the other preexisting taxes on the $X$s. Therefore the substitution of $t_W$ for $t_{Xj}$ will be welfare improving, over some range, on the basis of non-environmental considerations.

In addition, since $\text{MCPF}_W$ will rise with rising $t_W$, and that the $\text{MCPF}_X$ will rise as the tax rates on the $X$s are lowered, it follows directly that the magnitude of this welfare gain from substituting $t_W$ for $t_X$ will decline as the pollution tax rises.

If the optimal pollution tax is found to be greater than MED, then the two conditions just described are sufficient to conclude that the non-environmental benefits of an equal-yield environmental tax reform will be positive over some range. If a) the substitution of a pollution tax for preexisting taxes is initially welfare improving, and b) the size of that welfare gain declines with a rise in the pollution tax, then c) an optimal pollution tax $t^*_W > MED$, then as
illustrated in figure 3, there will necessarily be a tax range—in this case between zero and $t'_W$—where the non-environmental welfare effects will be positive.

The numerical model introduced above and described in Appendix A confirms this directly. We can compute the welfare changes for a Pigouvian tax where revenues are returned lump sum to the economy, and compare these for a ‘revenue neutral’ policy where the pollution tax revenues contribute to satisfying the budget constraint. The results in figure 2 are consistent with the inference drawn above, that using revenues from pollution taxes to cut distortionary taxes will raise welfare more than if they were simply returned lump sum to the economy. We can also see from figure 2 that the magnitude of the ‘second dividend’ rises with the marginal cost of public funds. This is because at any given level of tax $t_W$ the difference between $\text{MCPF}_W$ and $\text{MCPF}_X$ is larger the larger is the initial Ramsey tax.

C. Intuition When emissions taxes and waste disposal services are explicitly defined, the model provides a more intuitive way to understanding the existence of a double dividend. Any environmental service, whether it be waste disposal, recreational use, or resource extraction, represents a service flow from an exogenous environmental asset. In many such cases, however, the absence of secure property rights and markets for these environmental goods presents an opportunity for government to introduce a tax which, in addition to promoting allocative efficiency, will appropriate pure resource rents.

The appeal of appropriating the pure rents associated with goods or services in fixed supply is a well-known part of the contemporary literature on exhaustible resources rents (Gray 1914; Gaffney 1967; and Dasgupta and Heal 1979). Indeed, Henry George’s (1955) proposed "single tax," reflects this same idea, interpreted broadly as a tax on natural resources with wide applicability to modern environmental and resource problems (see Yandle and Barnett 1974; Whitaker 1997). Although the similarity between taxing rents from exhaustible resources and taxing rents from a less tangible resource such as "location" may not at first glance be obvious, they are both examples of assets where inelastic supply makes them eminently suitable for taxation—as would many other environmental services.
For an environmental waste sink with a fixed assimilative capacity, \( Q \), the associated pure rents can, in principle, be taxed away without distortion. By introducing a Pigouvian tax on waste disposal, \( W \), government accomplishes three things. First, it restores allocative efficiency so that \( \partial u / \partial w = m \left[ \partial u / \partial E \right] \left[ \partial e / \partial mZ \right] \). Second, it restores the rents associated with allocative efficiency (at the first-best allocation individual’s would be willing to pay MED for \( W \), and MED/\( m \) for a corresponding change in \( E \)). And third, the tax appropriates those rents from consumers of \( W \) (we assume that non-exclusivity prohibits contemplating charges for consumption of \( E \)). Because appropriation of the rents in this instance also restores rather than distorts allocative efficiency, the tax will reduce the overall social cost of the tax system if substituted for pre-existing revenue-motivated taxes.

Indeed, the correspondence of the double dividend theory with existing resource rent theory is direct. The traditional argument for taxing the extraction of exhaustible resources, or in George's case of land rents, involves a single welfare gain arising from the substitution of non-distorting for distorting taxes. Land and mineral resources do not generally suffer from property rights failures, and thus are assumed to be allocated efficiently both before and after the introduction of such a tax. The double dividend notion can be seen as an extension of this existing theory to a resource where property rights and coordination failures represent allocative inefficiency in addition to an opportunity for rent appropriation.

If these property rights and coordination failures are absent, then the efficient allocation of \( W \) and \( E \) is assumed to occur through market allocations with the associated welfare gains (relative to the market failure scenario) and a redistribution of income from net buyers of \( W \) and \( E \) to their respective net sellers. In this case the government’s tax problem in [20] is simplified since the solutions to the individual’s choice functions, \( X* \), \( W* \), and \( V* \) will include the condition

\[
\frac{p_e}{p_w} = m \frac{\partial u}{\partial e} \frac{\partial e}{\partial mZ}.
\]
The environmental tax problem vanishes, and there is no possibility of an environmental dividend since allocative efficiency is assured through the market.

We are still left, however, with the possibility of revenue-motivated taxes. First, we can consider taxing both $W$ and $E$ at $t_W = p_W$ and $t_E = p_E$. Since sellers (owners) of these endowed environmental goods incur no costs, these tax rates will be non-distorting, identical to the tax on pure rents from an exhaustible resource.

When all pure rents have been appropriated, it may still make sense to raise the tax on pollution higher so long as the marginal cost of revenue from this tax is lower than the other revenue-motivated taxes. The optimal pollution tax will be reached when the marginal cost of public funds for the pollution tax (net of its ancillary environmental benefits) is just equal to the marginal cost of public funds on other goods (Schöb 1996). Thus, the optimal tax on pollution will generally exceed the Pigouvian rate. This may not always be the case, however, for example if a tax equal to MED fall on the elastic portion of the demand curve.

These results are also relevant and applicable to many non-pollution environmental services and congestible public goods, including ocean fisheries where auctioning transferable fishing quotas would accomplish the same thing, or for congestion pricing on highways, recreation area access permits, broadcast rights to pieces of the electromagnetic spectrum, or water rights. Indeed this last example was the focus of one of the earliest suggestions that a renewable resource might be the source of pure rents and a double dividend (Tullock 1967).

VIII. Conclusions

The recent literature on environmental taxation has mistaken a normalization error for evidence of a new distortionary economic phenomenon. The analysis here finds that there is no

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9 We continue to assume that it is not feasible to tax the amenity benefits of the environment. For some cases where direct taxation may not be practical, optimal taxation may nevertheless be achieved through a set of indirect taxes on commodities or inputs that affect the amount of the externality produced (see Holtermann 1976). For example, differential taxation of energy sources (coal, gas, biomass, solar) according to their carbon emissions would be equivalent to a carbon tax.
new ‘tax interaction effect’ and that what appear to be incongruous results have arisen from comparing and combining monetary values expressed in different, and incompatible, units. The recent literature includes analytical models in which private income is normalized in net income units while public revenues are normalized in gross income units. Similarly, this literature contains analysis of numerical models for which the household budget is normalized in units of net income while marginal environmental damages are expressed in units of gross income. These oversights are equivalent to comparing optimal taxes in one currency with marginal environmental damages expressed in a different currency, and drawing conclusions directly.

When these oversights are corrected by re-normalizing the results, estimates for the US economy indicate that the optimal pollution tax should be set above marginal environmental damages and, by inference, that the welfare gains from an equal-yield environmental tax reform will be higher than what a Pigouvian analysis would suggest.

These incorrect results have been widely disseminated and interpreted in policy contexts. This is particularly unfortunate where the recent literature has included the claim that environmental tax reform in response to increased environmental concern “reduces employment” (Bovenberg and van der Ploeg 1994, p. 383). In addition to mistakenly rejecting the double dividend hypothesis due to incompatible normalizations, this claim somehow emerges from a model with perfect labor markets and full unemployment. This particular assertion has become politically influential in policy debates about environmental taxation, especially in Europe. The current analysis supports a very different interpretation; that an equal-yield environmental tax reform can be expected to raise welfare, improve efficiency, and contribute to overall economic competitiveness.

To restate the correct and positive findings regarding the double dividend hypothesis, the following conclusions can be drawn from the above analysis. First, for reasonable estimates of the marginal cost of public funds and associated tax rates, the optimal pollution tax will typically exceed marginal environmental damages when pollution taxes contribute to government revenues. Second, the magnitude of this premium will rise with an increase in the marginal cost
of public funds: that is, the optimal tax on a polluting good will generally rise faster than the optimal tax on non-polluting goods. Third, the collective good of environmental quality is a complement to, not a competitor with, the provision of other public goods. And fourth, unlike other public goods which must be built and whose optimal levels decline with a rise in the cost of public funds, the marginal cost of environmental policy declines, and the optimal level of environmental quality rises, with a rise in the marginal cost of public funds.

Estimates of the optimal pollution tax for typical goods in the US economy exceed marginal environmental damages by about 33 percent.
REFERENCES


Appendix A. The two-good model as a non-generalizability special case

Fullerton (1997) and Metcalf (1999) find that even with a standard normalization (setting the income tax to zero), the Bovenberg and de Mooij model (1994) produces an optimal pollution tax below the Pigouvian rate. Can such a result from a model with only two-goods can be generalized? The Bovenberg and de Mooij result implies that the Ramsey term in Sandmo’s optimal tax relation is less than one (see section V). The magnitude of the Ramsey term is a function of the uncompensated elasticity of labor supply, which is assumed to take a value between 0.10 and 0.20. However, for a given labor supply elasticity, the Ramsey term will be a function of the number of goods in the model so long as the cross-price effects are non-zero.

To show this, begin with a one-good model, denoting untaxed leisure as \( X_0 \) and the single commodity as \( X_1 \). The Ramsey term in this case is just equal to the inverse of the own-price elasticity of demand, or

\[
R_1 = \frac{-x_{1}}{p_{1}S_{11}}
\]

where the elements of the Slutsky matrix are \( S_{ij} = \frac{\partial x_{i}}{\partial p_{j}} \).

The assumption of a positive labor supply elasticity in a one-good world requires that the own-price elasticity of demand for \( X_1 \) is greater than one, in which case we can conclude that \( R < 1 \), and therefore that the optimal pollution tax will indeed lie below MED. However, this result contradicts the inference from the Bovenberg and Goulder model for which \( R = 2 \), as well as for nearly all estimates of the MCPF and their corresponding tax rates.

The results from a two-good model will not be generalizable if it can be shown that, under reasonable assumptions, the Ramsey term, \( R \), is an increasing function of the number of goods. We will demonstrate here that for plausible assumptions \( R \) may rise with the number of goods, so long as the cross-price effects are non-zero.
In a one-good model where $X_0$ is leisure, $X_1$ is the only commodity, $H$ is the time endowment, and $H-X_0$ is labor supply. Defining full income, $M = p_0H$, and assuming that all prices and quantities equal unity, the following conditions must hold:

\[
\begin{align*}
\varepsilon_{00} + \varepsilon_{01} + \varepsilon_{0M} &\equiv 0 \quad \text{[A2a]} \\
\varepsilon_{10} + \varepsilon_{11} + \varepsilon_{1M} &\equiv 0 \quad \text{[A2b]} \\
-p_0(H-X_0) + p_1X_1 & = 0 \quad \text{[A2c]} \\
k_1\varepsilon_{1m} + k_2\varepsilon_{2m} &\equiv 1 \quad \text{[A2d]}
\end{align*}
\]

where $(p_iX_i/M = k_i)$.

The conditions that must hold in an equivalent two-good model are

\[
\begin{align*}
\varepsilon_{00} + \varepsilon_{01} + \varepsilon_{02} + \varepsilon_{0M} &\equiv 0 \quad \text{[A3a]} \\
\varepsilon_{10} + \varepsilon_{11} + \varepsilon_{12} + \varepsilon_{1M} &\equiv 0 \quad \text{[A3b]} \\
\varepsilon_{20} + \varepsilon_{21} + \varepsilon_{22} + \varepsilon_{2M} &\equiv 0 \quad \text{[A3c]} \\
-p_0(H-X_0) + p_1X_1 + p_2X_2 & = 0 \quad \text{[A3d]} \\
k_1\varepsilon_{1m} + k_2\varepsilon_{2m} + k_3\varepsilon_{3m} &\equiv 1 \quad \text{[A3e]}
\end{align*}
\]

For the two-good case the Ramsey term is

\[
R^2 = \frac{[-X_1S_{22}] + S_{12}X_2}{[p_1S_{11}S_{22}] - S_{12}S_{21}p_1}.
\]

Comparing $R^2$ with $R^1$ we see that the ratio of bracketed terms in $R^2$ will equal $R^1$ if the components that occur in both cases are unchanged. The numerator of $R^2$ increases by $S_{12}X_2$ and the denominator is reduced by $p_1S_{12}S_{21}$. But are their plausible assumption under which the ratio of the bracketed terms can remain unaffected by the increase in dimensionality of the model?

Taking a convenient special case we will assume that

1) quantities are unchanged across models ($X_j^n = X_j^{n+1} = 1$)
2) own-price elasticities are equal across models ($\varepsilon_{ii}^n = \varepsilon_{ii}^{n+1}$).
3) own-price elasticities are equal for all commodities ($\varepsilon_{ii} = \varepsilon_{jj}$).
4) given restrictions on preferences we assume $\epsilon_{ij} = \epsilon_{ji}$ for all commodities

Given 1) and 2) it must follow from the budget constraint that for a model with $n$ goods (excluding leisure) that $p_0$ is unchanged, and that the following must hold:

5) $p_{jn}^n = 1/n$ for $i \neq 0$;

6) $S_{ii}^n = nS_{ii}$;

7) $\epsilon_{ij}^n = (1/n)\epsilon_{ij}^1$.

From these simplifying assumptions it is possible that $p_1S_{11}$ remains unchanged between $R^n$ and $R^{n+1}$, and thus that the ratio of the bracketed terms in $R^2$ can remain constant so that $R^2 > R^1$.

Given our assumptions, the magnitude of the increase in $R$ with an increase in $n$ depends on the magnitude of $S_{12}$ and $S_{21}$. Indeed, if we were to assume independent demands, then the cross-price effects would be zero, the Ramsey term would be independent of the number of goods, and our assumption that labor supply is positively sloped would require a Ramsey term less than one independent of the number of goods in the model.

However, for non-zero cross-price terms, the larger is $S_{12}$ (and $S_{21}$) the greater will be the increase in the numerator, and the decrease in the denominator. For the specific case under scrutiny: $\epsilon_{11}^2 = \epsilon_{11}^1$, all $\epsilon_{ij}$s are equal where $i \neq j$, $\epsilon_{iM}$ is independent of $n$, and $\epsilon_{12}^2 = \frac{1}{2}(\epsilon_{10}^1)$. The value of $\epsilon_{10}^1$ will be larger the greater is the value of $\epsilon_{11}^1$, and the smaller is $\epsilon_{1M}^1$. Thus, going from a one-good to a two-good model will increase the Ramsey term as an increasing function of the own-price elasticity of demand and a decreasing function of the income elasticity for commodities.

This analytical exercise can be extended to $n=3$, with similar but tedious results. In lieu of this, a numerical example is given here where for the one-good model $\epsilon_{11} = -1.5$, and $\epsilon_{10} = 1$. Expanding the number of goods holding the conditions specified above gives us Ramsey terms $R^1 = 0.667$, $R^2 = 1.0$, and $R^3 = 1.2$.

Based on more general models, or based on empirical estimates of MCPF and tax rates, a value of $R = 2$ appears to be a more realistic assumption.
Appendix B. Specific model structure and parameter values for the numerical model

The numerical model has a CES utility function with separable arguments for leisure (V) (where $H - L = V$), and the environment (E) given as

$$U^i = (X^{-v} + Z^{-v})^{-k/v} + aV^g + E$$

Environmental quality is a function of the disposal of waste into the environment,

$$E = E_0 - (q \cdot Z^i) n .$$

Individuals maximize utility $U^i$ subject to labor income, Y, and expenditure constraint

$$(1 + t_x) X + (1 + t_z) Z = M^i = (H - L^i) .$$

The government’s optimal tax problem is to satisfy a given revenue requirement R with taxes on commodities and pollution so that

$$R = t_x X + t_z Z .$$

The following parameter values were assumed for the model:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>values</th>
</tr>
</thead>
<tbody>
<tr>
<td>k</td>
<td>.50</td>
</tr>
<tr>
<td>g</td>
<td>.45</td>
</tr>
<tr>
<td>v</td>
<td>1</td>
</tr>
<tr>
<td>E_0</td>
<td>0</td>
</tr>
<tr>
<td>q</td>
<td>0.02575</td>
</tr>
<tr>
<td>H</td>
<td>100</td>
</tr>
<tr>
<td>n</td>
<td>1</td>
</tr>
</tbody>
</table>

The value of q is chosen so that $MED \equiv q/\lambda = 1$ for the first-best situation where $\lambda$ is the marginal utility of income. MED will not equal one with non-zero Ramsey taxes. The values of k and g are chosen to calibrate the model so that the compensated labor supply elasticities are similar to those estimated in the literature.
Table 1. Estimates of optimal pollution taxes for the US economy: comparing numerical simulation results for alternative normalizations with estimates based on Sandmo’s optimal tax formula

<table>
<thead>
<tr>
<th>Model version and parameters</th>
<th>Results from Bovenberg and Goulder (1996)</th>
<th>Evaluation using Sandmo’s formula (1975)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case</td>
<td>Income tax rate$^a$</td>
<td>MCPF$^a$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. Central cases:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>.40</td>
<td>.20</td>
<td>.60</td>
</tr>
<tr>
<td>.20</td>
<td>.40</td>
<td>67</td>
</tr>
<tr>
<td>.60</td>
<td>.40</td>
<td>53</td>
</tr>
<tr>
<td>2. Varying intertemporal elasticity of substitution:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>low</td>
<td>.40</td>
<td>63</td>
</tr>
<tr>
<td>high</td>
<td>.40</td>
<td>53</td>
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<td>3. Varying labor supply elasticity:</td>
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<tr>
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<tr>
<td>high</td>
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<td>54</td>
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<td>4. Varying energy substitution elasticity:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>low</td>
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<td>61</td>
</tr>
<tr>
<td>high</td>
<td>.40</td>
<td>55</td>
</tr>
</tbody>
</table>

$^a$ From Bovenberg and Goulder (1996), table 3, page 994 where MED is assumed to be $75 (when valued in units of gross income as explained in section IV above).

$^b$ The net income measure of MED will be the gross income figure times (1-\(t_L\)), or $45 for the base case where \(t_L = .40\).

$^c$ Parameters for each case (the income tax rate and MCPF) are applied to equations [22] and [23] in section V above.
Figure 1. Optimal pollution taxes as a share of marginal environmental damages: numerical simulation model results for alternative normalizations.
Figure 2. Welfare effects from environmental taxation: numerical simulation model results
Figure 3. Non-environmental benefits of environmental tax reform